1. Determine a recursive definition for each of the following sequences.
   a) \( a_n = 2n - 3 \quad (n = 0, 1, 2, \ldots) \).
   b) \( b_n = \frac{2^{n-1}}{3} \quad (n = 1, 2, 3, \ldots) \).

2. For each of the two sequences:
   - \( a_{n+1} = a_n + 5 \quad (n = 0, 1, 2, \ldots) \) where \( a_0 = -14 \),
   - \( b_n = 3b_{n-1} \quad (n = 2, 3, 4, \ldots) \) where \( b_1 = 2 \).
   a) Write down a closed form for the sequence.
   b) Calculate the 10th term of the sequence.
   c) Determine which term of the sequence would be the first to exceed 1000.

3. Let \( \{F_n\} \) be the Fibonacci sequence defined in the course notes.
   a) For each value of \( n = 2, 3, 4, 5, 6 \), calculate the value of \( F_{n-1}F_{n+1} - F_n^2 \).
   b) Based on the pattern observed in part (a), write down a guess for a formula for \( F_{n-1}F_{n+1} - F_n^2 \) in terms of \( n \). (You do not have to prove that your formula is correct, but it must work for the values of \( n \) that you calculated in part (a).)

4. Determine the value of each of the following series.
   a) \( \sum_{i=1}^{50} 3n - 5 \).
   b) \( \sum_{i=4}^{35} -4n + 100 \).
   c) \( \sum_{i=0}^{10} -3 \times 2^i \).
   d) \( \sum_{i=1}^{\infty} (-4)^i \).
   e) \( \sum_{i=3}^{\infty} \left(\frac{1}{3}\right)^i \).

(Please turn over.)
5. In each of the following, the first six terms of a sequence whose closed form is a polynomial are listed. Use the method of finite differences to determine a closed form for the corresponding sequence.

   a) 3, 7, 13, 21, 31, 43.
   b) 10, 3, −4, −11, −18, −25.
   c) −1, −3, 1, 17, 51, 109.

6. Use the method of telescoping sums to determine a closed form for each of the following sequences.

   a) \(a_{n+1} = a_n + 12(4^{n-1})\) \((n = 1, 2, 3, \ldots)\) where \(a_1 = 1\).
   b) \(b_{n+1} = b_n + 2n\) \((n = 1, 2, 3, \ldots)\) where \(b_1 = 4\).

7. You invest $10,000 in a term deposit with an interest rate of 6% per year compounded quarterly. What is the value of your term deposit after 5 years?

8. How much do you have to invest in a term deposit paying interest of 5% per year compounded monthly to provide $20,000 in 10 years time?

9. A population of squirrels numbered 50 on 30 June 1980. On 30 June 1990, the population of squirrels numbered 60. The population can be modelled using the exponential population model.

   a) Determine the yearly growth rate of this population of squirrels.
   b) Estimate the squirrel population on 30 June 1985.

10. Use mathematical induction to prove the following statements.

   a) \(\sum_{i=1}^{n} i^3 = \frac{n^2(n + 1)^2}{4}\).
   b) \(\sum_{i=1}^{n} \frac{1}{i(i + 1)} = \frac{n}{n + 1}\).
   c) For all \(n \geq 1\), 5 divides \(6^n - 1 + 5n\).