3 Solving equations

3.1 Introduction

Solving equations involves rearranging and manipulating the terms in the equation until you have isolated the variable for which you are solving. To manipulate equations you must do the same operation to both sides of the equation. For example, consider $3x = 12$. If we divide the left-hand side by 3, we must also divide the right-hand side by 3 in order for the statement to remain true. Thus $3x = 12$ is equivalent to $3x \div 3 = 12 \div 3$ which is equivalent to $x = 4$. We have now isolated the variable x and we can see that the solution is $x = 4$.

3.2 Solving linear equations

Here are some examples of linear equations.

$$20 - x = 12 \qquad y = 5x - 6 \qquad \frac{3a}{8} = 16$$

A linear equation is an equation in which any variable that appears must be to the power of one, and there are no products like xy. A linear equations containing one or two variables has a graph that is a straight line; refer to Section 9 for information on graphing lines. If a linear equation has only one unknown, we can solve for that unknown.

Example *3.2.1* Solve $18 - x = 6$.

Solution:

$$\begin{aligned} 18 - x &= 6 \\ 18 - x - 18 &= 6 - 18 \\ -x &= -12 \\ -x \times (-1) &= -12 \times (-1) \\ x &= 12 \end{aligned}$$

The solution to $18 - x = 6$ is $x = 12$.

When writing out your solution it is not necessary to write out all the intermediate operations. Just write the result of each operation. Here is how a typical solution might look.

Solution:

$$\begin{aligned} 18 - x &= 6 \\ -x &= -12 \\ x &= 12 \end{aligned}$$

When solving equations with fractions, it is usually a good idea to first multiply both sides of the equation by a number that will get rid of any fractions.

Example *3.2.2* Solve $\dfrac{3x+6}{12} = 10$.

Solution: We start by multiplying both sides by 12.

$$\begin{aligned} 3x+6 &= 120 && \text{now subtract 6 from both sides} \\ 3x &= 114 && \text{now divide both sides by 3} \\ x &= 38 \end{aligned}$$

The solution to $\dfrac{3x+6}{12} = 10$ is $x = 38$.

Example *3.2.3* Solve $\dfrac{5y}{8} + 1 = \dfrac{y}{3}$.

Solution: We start by multiplying both sides by 24.

$$\begin{aligned} 15y+24 &= 8y && \text{now subtract 24 and } 8y \text{ from both sides} \\ 7y &= -24 && \text{now divide both sides by 7} \\ y &= -\frac{24}{7} \end{aligned}$$

The solution to $\dfrac{5y}{8} + 1 = \dfrac{y}{3}$ is $y = -\dfrac{24}{7}$.

Example *3.2.4* Solve $\dfrac{9}{15x} = 12$.

Solution:

$$\begin{aligned} \frac{9}{15x} &= 12 && \text{now multiply both sides by } 15x \\ 9 &= 180x && \text{now divide both divide both sides by 180} \\ \frac{1}{20} &= x \end{aligned}$$

The solution to $\dfrac{9}{15x} = 12$ is $x = \dfrac{1}{20}$.

3.3 Relationships between variables

When you come across equations that have two variables, you may need to express one variable in terms of the other. This is common when graphing functions, where you often express y in terms of x. To find one variable in terms of the other, isolate the variable you require by rearranging the equation.

Example *3.3.1* Given $x = 3y + 21$, express y in terms of x.

Solution:

$$\begin{aligned} x &= 3y+21 \\ x-21 &= 3y \\ \frac{x}{3} - 7 &= y \end{aligned}$$

The solution is $y = \dfrac{x}{3} - 7$.

Example *3.3.2* Given $\frac{4a}{7} = 2b$, express a in terms of b.

Solution:

$$\begin{aligned} \frac{4a}{7} &= 2b \\ 4a &= 14b \\ a &= \frac{7b}{2} \end{aligned}$$

The solution is $a = \frac{7b}{2}$.

Example *3.3.3* Given $\frac{9}{8x} = \frac{2y}{3}$, express x in terms of y and then y in terms of x $(x, y \neq 0)$.

Solution:

$$\begin{aligned} \frac{9}{8x} &= \frac{2y}{3} \\ 9 &= \frac{16xy}{3} \\ 27 &= 16xy \\ \frac{27}{16y} &= x \end{aligned}$$

$$\begin{aligned} \frac{9}{8x} &= \frac{2y}{3} \\ \frac{27}{8x} &= 2y \\ \frac{27}{16x} &= y \end{aligned}$$

The solutions are $x = \frac{27}{16y}$ and $y = \frac{27}{16x}$.

Practice Problems

Here are some problems for you to practice on, followed by answers. Fully worked solutions to these problems can be found in Section 11.3.

Q3.1 Solve the following equations.

- **(a)** $13 = \dfrac{4a-5}{3}$
- **(b)** $\dfrac{7y}{9} - 6 = 5$
- **(c)** $\dfrac{8}{6x} = 3$
- **(d)** $\dfrac{11b}{12} = \dfrac{3}{4}$

Q3.2 Given the following equations, express a in terms of b.

- **(a)** $\dfrac{14a}{17} = 2b$
- **(b)** $\dfrac{3a}{4} = b + 6$
- **(c)** $\dfrac{28}{4a} = \dfrac{b}{5}$ $(a, b \neq 0)$
- **(d)** $\dfrac{12}{3a} = \dfrac{4}{7b}$ $(a, b \neq 0)$

A3.1 **(a)** $a = 11$

- **(b)** $y = \dfrac{99}{7}$
- **(c)** $x = \dfrac{4}{9}$
- **(d)** $b = \dfrac{9}{11}$

A3.2 **(a)** $a = \dfrac{17b}{7}$

- **(b)** $a = \dfrac{4b}{3} + 8$
- **(c)** $a = \dfrac{35}{b}$
- **(d)** $a = 7b$