11 Practice problem solutions

This section gives worked solutions to all of the practice problems from Sections 1 to 10.

11.1 Factorisation

A1.1 **(a)** $4y^2 - 16y = 4y \times y - 4y \times 4 = 4y(y-4)$

(b) $32ab + 16b + 8abc = 8b \times 4a + 8b \times 2 + 8b \times ac = 8b(4a + 2 + ac)$

(c) $6xyz + 3yz + 18wyz = 3yz \times 2x + 3yz \times 1 + 3yz \times 6w = 3yz(2x + 1 + 6w)$

A1.2 **(a)** $4 + 4b + b^2 = (2+b)(2+b) = (2+b)^2$

(b) $25a^2 - 10ad + d^2 = (5a-d)(5a-d) = (5a-d)^2$

(c) $x^2 + 12xy + 36y^2 = (x+6y)(x+6y) = (x+6y)^2$

(d) $144 - 48a + 4a^2 = (12-2a)(12-2a) = (12-2a)^2$

A1.3 **(a)** $9y^2 - 36 = (3y)^2 - 6^2 = (3y-6)(3y+6)$

(b) $64 - 121g^4 = 8^2 - (11g^2)^2 = (8 - 11g^2)(8 + 11g^2)$

(c) $16x^6 - 49y^8 = (4x^3)^2 - (7y^4)^2 = (4x^3 - 7y^4)(4x^3 + 7y^4)$

A1.4 **(a)**
$$\begin{aligned} & x^2 - 5x + 4 \\ = \; & x^2 - 4x - 1x + 4 \\ = \; & x(x-4) - 1(x-4) \\ = \; & (x-4)(x-1) \end{aligned}$$

(b)
$$\begin{aligned} & x^2 + 2x - 15 \\ = \; & x^2 - 3x + 5x - 15 \\ = \; & x(x-3) + 5(x-3) \\ = \; & (x-3)(x+5) \end{aligned}$$

(c)
$$\begin{aligned} & y^2 - 3y + 2 \\ = \; & y^2 - 2y - 1y + 2 \\ = \; & y(y-2) - 1(y-2) \\ = \; & (y-2)(y-1) \end{aligned}$$

(d)
$$\begin{aligned} & x^2 + 13x + 42 \\ = \; & x^2 + 7x + 6x + 42 \\ = \; & x(x+7) + 6(x+7) \\ = \; & (x+7)(x+6) \end{aligned}$$

(e)
$$\begin{aligned} & x^2 - 11x + 24 \\ = \; & x^2 - 8x - 3x + 24 \\ = \; & x(x-8) - 3(x-8) \\ = \; & (x-8)(x-3) \end{aligned}$$

(f)
$$\begin{aligned} & a^2 + 7a + 12 \\ = \; & a^2 + 4a + 3a + 12 \\ = \; & a(a+4) + 3(a+4) \\ = \; & (a+4)(a+3) \end{aligned}$$

A1.5 **(a)**

$$\begin{aligned} & 3x^2 + 17x + 10 \\ = \; & 3x^2 + 15x + 2x + 10 \\ = \; & 3x(x+5) + 2(x+5) \\ = \; & (x+5)(3x+2) \end{aligned}$$

(b)

$$\begin{aligned} & 2y^2 + 12y + 16 \\ = \; & 2y^2 + 8y + 4y + 16 \\ = \; & 2y(y+4) + 4(y+4) \\ = \; & (y+4)(2y+4) \end{aligned}$$

(c)

$$\begin{aligned} & 4x^2 - 8x - 12 \\ = \; & 4x^2 - 12x + 4x - 12 \\ = \; & 2x(2x-6) - 2(2x-6) \\ = \; & (2x-6)(2x-2) \end{aligned}$$

or

$$\begin{aligned} & 4x^2 - 8x - 12 \\ = \; & 4x^2 - 12x + 4x - 12 \\ = \; & 4x(x-3) - 4(x-3) \\ = \; & (x-3)(4x+4) \end{aligned}$$

or

$$\begin{aligned} & 4x^2 - 8x - 12 \\ = \; & 4x^2 - 12x + 4x - 12 \\ = \; & x(4x-12) + 1(4x-12) \\ = \; & (4x-12)(x+1) \end{aligned}$$

(d)

$$\begin{aligned} & 2z^2 - z - 15 \\ = \; & 2z^2 - 6z + 5z - 15 \\ = \; & 2z(z-3) + 5(z-3) \\ = \; & (z-3)(2z+5) \end{aligned}$$

11.2 Manipulating fractions

A2.1 (a) $\frac{14x}{18x} = \frac{14x \div 2x}{18x \div 2x} = \frac{7}{9}$

(b) $\frac{9ab}{27bc} = \frac{9ab \div 9b}{27bc \div 9b} = \frac{a}{3c}$

(c) $\frac{8 + 4d}{2e} = \frac{2(4 + 2d) \div 2}{2e \div 2} = \frac{4 + 2d}{e}$

A2.2 (a) $\frac{3}{12} + \frac{1}{4} = \frac{3}{12} + \frac{3}{12} = \frac{6}{12} = \frac{1}{2}$

(b) $\frac{1}{3} + \frac{4}{15} = \frac{5}{15} + \frac{4}{15} = \frac{9}{15} = \frac{3}{5}$

(c) $\frac{6}{7} - \frac{2}{3} = \frac{18}{21} - \frac{14}{21} = \frac{4}{21}$

(d) $\frac{13}{18} - \frac{7}{9} = \frac{13}{18} - \frac{14}{18} = -\frac{1}{18}$

A2.3 (a) $\frac{4}{11} \times \frac{7}{12} = \frac{28}{132} = \frac{7}{33}$

(b) $\frac{3}{13} \div \frac{8}{9} = \frac{3}{13} \times \frac{9}{8} = \frac{27}{104}$

(c) $\frac{5}{8} \times \frac{2}{7} = \frac{10}{56} = \frac{5}{28}$

(d) $\frac{3}{8} \div \frac{5}{6} = \frac{3}{8} \times \frac{6}{5} = \frac{18}{40} = \frac{9}{20}$

A2.4 (a) $\frac{ab}{4} + \frac{2c}{8d} = \frac{2abd}{8d} + \frac{2c}{8d} = \frac{2(abd + c)}{8d} = \frac{abd + c}{4d}$

(b) $\frac{4xy}{9} + \frac{3x}{12} = \frac{16xy}{36} + \frac{9x}{36} = \frac{x(16y + 9)}{36}$

(c) $\frac{30gh}{14} - \frac{5h}{7g} = \frac{30g^2h}{14g} - \frac{10h}{14g} = \frac{10h(3g^2 - 1)}{14g} = \frac{5h(3g^2 - 1)}{7g}$

(d) $\frac{9a}{24} - \frac{5}{8b} = \frac{9ab}{24b} - \frac{15}{24b} = \frac{9ab - 15}{24b} = \frac{3ab - 5}{8b}$

A2.5 (a) $\frac{12g}{16h} \times \frac{h}{2} = \frac{12gh}{32h} = \frac{3g}{8}$

(b) $\frac{4e}{45} \times \frac{9d}{f} = \frac{36ed}{45f} = \frac{4ed}{5f}$

(c) $\frac{xy}{7} \div \frac{x}{14} = \frac{xy}{7} \times \frac{14}{x} = \frac{14xy}{7x} = 2y$

(d) $\frac{5a}{7} \div \frac{7}{9a} = \frac{5a}{7} \times \frac{9a}{7} = \frac{45a^2}{49}$

11.3 Solving Equations

A3.1 **(a)**
$$\begin{aligned} 13 &= \frac{4a-5}{3} \\ 39 &= 4a-5 \\ 44 &= 4a \\ 11 &= a \end{aligned}$$
The solution to $13 = \frac{4a-5}{3}$ is $a = 11$.

(b)
$$\begin{aligned} \frac{7y}{9} - 6 &= 5 \\ 7y - 54 &= 45 \\ 7y &= 99 \\ y &= \frac{99}{7} \end{aligned}$$
The solution to $\frac{7y}{9} - 6 = 5$ is $y = \frac{99}{7}$.

(c)
$$\begin{aligned} \frac{8}{6x} &= 3 \\ 8 &= 18x \\ \frac{8}{18} &= x \\ \frac{4}{9} &= x \end{aligned}$$
The solution to $\frac{8}{6x} = 3$ is $x = \frac{4}{9}$.

(d)
$$\begin{aligned} \frac{11b}{12} &= \frac{3}{4} \\ 11b &= 9 \\ b &= \frac{9}{11} \end{aligned}$$
The solution to $\frac{11b}{12} = \frac{3}{4}$ is $b = \frac{9}{11}$.

A3.2 **(a)**
$$\begin{aligned} \frac{14a}{17} &= 2b \\ 14a &= 34b \\ a &= \frac{34b}{14} \\ a &= \frac{17b}{7} \end{aligned}$$
The solution is $a = \frac{17b}{7}$.

(b)
$$\begin{aligned} \frac{3a}{4} &= b + 6 \\ 3a &= 4b + 24 \\ a &= \frac{4b}{3} + 8 \end{aligned}$$
The solution is $a = \frac{4b}{3} + 8$.

(c)

$$\begin{aligned}\frac{28}{4a} &= \frac{b}{5}\\ 28 &= \frac{4ab}{5}\\ 140 &= 4ab\\ \frac{140}{4b} &= a\\ \frac{35}{b} &= a\end{aligned}$$

The solution is $a = \frac{35}{b}$.

(d)

$$\begin{aligned}\frac{12}{3a} &= \frac{4}{7b}\\ 12 &= \frac{12a}{7b}\\ 84b &= 12a\\ 7b &= a\end{aligned}$$

The solution is $a = 7b$.

11.4 Powers

A4.1 9^5 written as a product is $9 \times 9 \times 9 \times 9 \times 9$.

A4.2 **(a)** $x^2 \times x^3 = x^{2+3} = x^5$

(b) $\dfrac{y^5}{y^2} = y^{5-2} = y^3$

(c) $(x^4)^3 = x^{4\times3} = x^{12}$

(d) $(2x)^3 = 2^3 \times x^3 = 8x^3$

(e) $x^{-3} = \dfrac{1}{x^3}$

A4.3 **(a)** $\dfrac{2^{13} \times 2^4}{2^{14}} = \dfrac{2^{13+4}}{2^{14}} = \dfrac{2^{17}}{2^{14}} = 2^{17-14} = 2^3 = 8$

(b) $125^{2/3} = (125^{1/3})^2 = ((5^3)^{1/3})^2 = (5^{3\times1/3})^2 = (5^1)^2 = 25$

(c) $(x \times y)^0 = x^0 \times y^0 = 1 \times 1 = 1$

A4.4 $\dfrac{(x^7 \times y^{-4})^2}{(y^{12} \times x^3)^{-3}} = \dfrac{x^{7\times2} \times y^{-4\times2}}{y^{12\times-3} \times x^{3\times-3}} = \dfrac{x^{14} \times y^{-8}}{y^{-36} \times x^{-9}} = x^{14-(-9)} \times y^{-8-(-36)} = x^{23} \times y^{28}$

A4.5 $\sqrt{\dfrac{(x^{-3} \times y)^{-2}}{x^4 \times y^{-6}}} = \sqrt{\dfrac{x^{-3\times-2} \times y^{-2}}{x^4 \times y^{-6}}} = \sqrt{\dfrac{x^6 \times y^{-2}}{x^4 \times y^{-6}}} = \sqrt{x^{6-4} \times y^{-2-(-6)}} = \sqrt{x^2 \times y^4}$

$= (x^2 \times y^4)^{1/2} = x^{2\times1/2} \times y^{4\times1/2} = x \times y^2$ (provided $x > 0$ and $y \neq 0$)

A4.6 $\dfrac{2^{14} \times 9^{-3} \times 5^{-1}}{25 \times 8^2 \times 3^{-11}} = \dfrac{2^{14} \times (3^2)^{-3} \times 5^{-1}}{5^2 \times (2^3)^2 \times 3^{-11}} = \dfrac{2^{14} \times 3^{2\times-3} \times 5^{-1}}{5^2 \times 2^{3\times2} \times 3^{-11}} = \dfrac{2^{14} \times 3^{-6} \times 5^{-1}}{5^2 \times 2^6 \times 3^{-11}}$

$= 2^{14-6} \times 3^{-6-(-11)} \times 5^{-1-2} = 2^8 \times 3^5 \times 5^{-3}$

11.5 Trigonometry

A5.1 (a)

$$120 \times \frac{\pi}{180}$$
$$= \frac{120\pi}{180}$$
$$= \frac{2\pi}{3}$$

Thus 120° equals $\frac{2\pi}{3}$ radians.

(b)

$$260 \times \frac{\pi}{180}$$
$$= \frac{260\pi}{180}$$
$$= \frac{13\pi}{9}$$

Thus 260° equals $\frac{13\pi}{9}$ radians.

(c)

$$315 \times \frac{\pi}{180}$$
$$= \frac{315\pi}{180}$$
$$= \frac{7\pi}{4}$$

Thus 315° equals $\frac{7\pi}{4}$ radians.

A5.2 (a)

$$\frac{4\pi}{9} \times \frac{180}{\pi}$$
$$= \frac{720}{9}$$
$$= 80$$

Thus $\frac{4\pi}{9}$ radians 80°.

(b)

$$\frac{3\pi}{5} \times \frac{180}{\pi}$$
$$= \frac{540}{5}$$
$$= 108$$

Thus $\frac{3\pi}{5}$ radians 108°.

(c)

$$\frac{\pi}{9} \times \frac{180}{\pi}$$
$$= \frac{180}{9}$$
$$= 20$$

Thus $\frac{\pi}{9}$ radians 20°.

A5.3 **(a)** $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, so $\sin\theta = \frac{5}{13}$

(b) $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, so $\cos\theta = \frac{12}{13}$

(c) $\tan\theta = \frac{\text{opp}}{\text{adj}}$, so $\tan\theta = \frac{5}{12}$

A5.4 **(a)**

$$\begin{aligned} \cos 56^\circ &= \frac{x}{19} \\ x &= 19 \times \cos 56^\circ \\ x &= 10.62 \end{aligned}$$

The length of side x is 10.62cm.

(b)

$$\begin{aligned} \sin 15^\circ &= \frac{8}{z} \\ z \times \sin 15^\circ &= 8 \\ z &= \frac{8}{\sin 15^\circ} \\ z &= 30.91 \end{aligned}$$

The length of side z is 30.91cm.

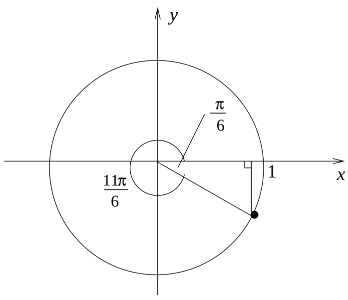
(c)

$$\begin{aligned} \tan\theta &= \frac{13}{12} \\ \theta &= \tan^{-1}\left(\frac{13}{12}\right) \\ \theta &= 47.29^\circ \end{aligned}$$

The angle θ is 47.29°.

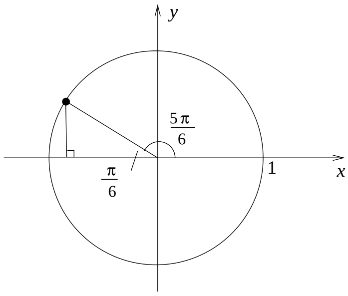
A5.5 **(a)**

$$\begin{aligned} \sin\frac{11\pi}{6} &= -\sin\frac{\pi}{6} \\ &= -\frac{1}{2} \end{aligned}$$

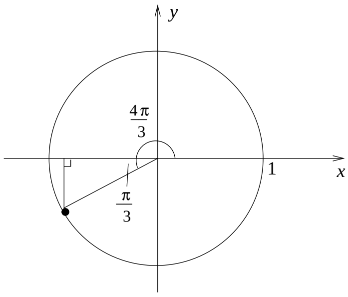


(b)

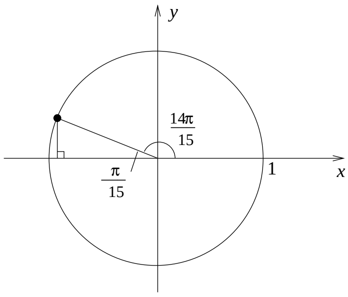
$$\begin{aligned} \cos\frac{5\pi}{6} &= -\cos\frac{\pi}{6} \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$



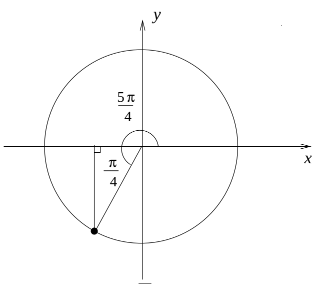
(c) $$\tan\frac{4\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$$

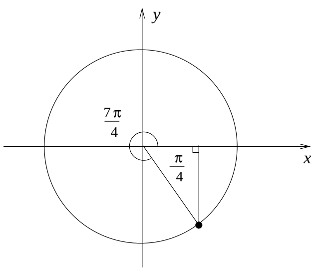


(d) $$\sin\frac{14\pi}{15} = \sin\frac{\pi}{15} = 0.208$$



A5.6 **(a)** Sine is negative in the 3rd and 4th quadrants. The angle in the 1st quadrant whose sine is $\frac{1}{\sqrt{2}}$ is $\frac{\pi}{4}$. Thus,

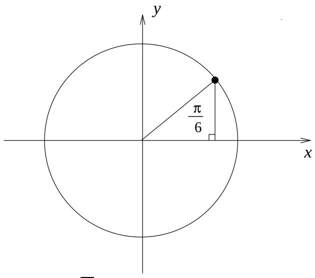


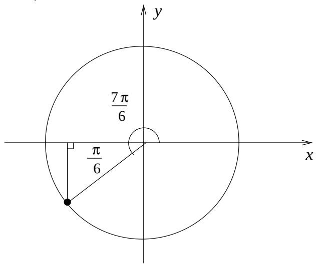


$$\theta = \pi + \frac{\pi}{4} \quad \text{and} \quad \theta = 2\pi - \frac{\pi}{4}$$
$$\theta = \frac{5\pi}{4} \quad \text{and} \quad \theta = \frac{7\pi}{4}.$$

(b) We know $\cos\theta = x$, and x is equal to 1 at the point (1,0), which represents the angles 0 and 2π.

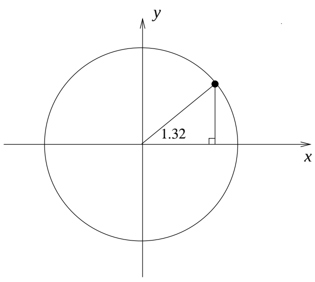
(c) Tangent is positive in the 1st and 3rd quadrants.The angle in the 1st quadrant whose tangent is $\frac{1}{\sqrt{3}}$ is $\frac{\pi}{6}$. Thus,

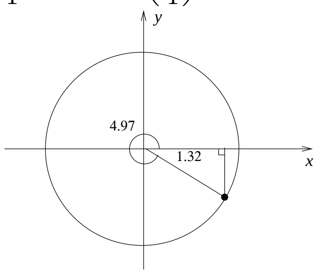




$$\theta = \frac{\pi}{6} \quad \text{and} \quad \theta = \pi + \frac{\pi}{6}$$
$$\theta = \frac{\pi}{4} \quad \text{and} \quad \theta = \frac{7\pi}{6}.$$

(d) Cosine is positive in the 1st and 4th quadrants.The angle in the 1st quadrant whose cosine is $\frac{1}{4}$ is $\cos^{-1}\left(\frac{1}{4}\right) = 1.32$ radians. Thus,





$$\theta = 1.32 \quad \text{and} \quad \theta = 2\pi - 1.32$$
$$\theta = 1.32 \quad \text{and} \quad \theta = 4.97$$

11.6 Solving quadratic equations

A6.1 **(a)** Yes, $x^2 - \frac{1}{2}x + 4 = 0$

(b) No, this is a cubic equation.

(c) Yes, this expands to $x^2 + x - 6 = 0$

(d) Yes, this is equivalent to $x^2 - 3x - 2 = 0$

A6.2 **(a)**

$$\begin{aligned} 2x^2 + 10x + 12 &= 0 \\ (x+2)(2x+6) &= 0 \\ x + 2 = 0 \quad &\text{or} \quad 2x + 6 = 0 \\ x = -2 \quad &\text{or} \quad 2x = -6 \\ x = -2 \quad &\text{or} \quad x = -3 \end{aligned}$$

The solutions to $2x^2 + 10x + 12 = 0$ are $x = -2$ and $x = -3$.

(b)

$$\begin{aligned} 4x^2 - 25 &= 0 \\ (2x)^2 - 5^2 &= 0 \\ (2x-5)(2x+5) &= 0 \\ 2x - 5 = 0 \quad &\text{or} \quad 2x + 5 = 0 \\ 2x = 5 \quad &\text{or} \quad 2x = -5 \\ x = \frac{5}{2} \quad &\text{or} \quad x = -\frac{5}{2} \end{aligned}$$

The solutions to $4x^2 - 25 = 0$ are $x = 5/2$ and $x = -5/2$.

(c)

$$\begin{aligned} x^2 + 4x &= 0 \\ x(x+4) &= 0 \\ x = 0 \quad &\text{or} \quad x + 4 = 0 \\ x = 0 \quad &\text{or} \quad x = -4 \end{aligned}$$

The solutions to $x^2 + 4x = 0$ are $x = 0$ and $x = -4$.

(d)

$$\begin{aligned} y^2 + 6y + 9 &= 0 \\ (y+3)(y+3) &= 0 \\ y + 3 &= 0 \\ y &= -3 \end{aligned}$$

The solution to $y^2 + 6y + 9 = 0$ is $y = -3$.

A6.3 **(a)**

$$\begin{aligned} x^2 + 6x + 9 &= 0 \\ x &= \frac{-6 \pm \sqrt{36 - 36}}{2} \\ x &= \frac{-6 \pm \sqrt{0}}{2} \\ x &= -3 \end{aligned}$$

The solution to $x^2 + 6x + 9 = 0$ is $x = -3$.

(b)
$$\begin{aligned} 7p^2 + 6p + 1 &= 0 \\ p &= \frac{-6 \pm \sqrt{36 - 28}}{14} \\ p &= \frac{-6 \pm \sqrt{8}}{14} \\ p &= \frac{-6 \pm 2\sqrt{2}}{14} \\ p &= \frac{-3 \pm \sqrt{2}}{7} \end{aligned}$$

$$p = \frac{-3 + \sqrt{2}}{7} \quad \text{or} \quad x = \frac{-3 - \sqrt{2}}{7}$$

The solutions to $7p^2 + 6p + 1 = 0$ are $p = \frac{-3 + \sqrt{2}}{7}$ and $p = \frac{-3 - \sqrt{2}}{7}$.

(c)
$$\begin{aligned} 2x^2 - 10x + 13 &= 0 \\ x &= \frac{10 \pm \sqrt{100 - 104}}{4} \\ x &= \frac{10 \pm \sqrt{-4}}{4} \end{aligned}$$

There are no real solutions.

11.7 Solving inequalities

A7.1 **(a)**
$$\begin{aligned} \frac{4x}{7} &\geq 3+x \\ 4x &\geq 21+7x \\ -3x &\geq 21 \\ x &\leq -7 \end{aligned}$$

The solution to $\frac{4x}{7} \geq 3+x$ is $x \leq -7$.

(b)
$$\begin{aligned} 12-6x &< 8-x \\ -5x &< -4 \\ x &> \frac{4}{5} \end{aligned}$$

The solution to $12-6x < 8-x$ is $x > \frac{4}{5}$.

(c)
$$\begin{aligned} \frac{11a+4}{6} &> 3+2a \\ 11a+4 &> 18+12a \\ -a &> 14 \\ a &< -14 \end{aligned}$$

The solution to $\frac{11a+4}{6} > 3+2a$ is $a < -14$.

(d) $\frac{7b-14}{b} \leq 5$

If $b > 0, b \neq 0$,
$$\begin{aligned} 7b-14 &\leq 5b \\ 2b &\leq 14 \\ b &\leq 7 \end{aligned}$$

If $b < 0$,
$$\begin{aligned} 7b-14 &\geq 5b \\ 2b &\geq 14 \\ b &\geq 7 \end{aligned}$$

Now, for $b < 0$ we have $b \geq 7$. Hence there is no solution when $b < 0$.

The solution to $\frac{7b-14}{b} \leq 5$ is $b > 0$ and $b \leq 7$, that is $0 < b \leq 7$.

A7.2 **(a)** $|3x| > 12$

If $3x \geq 0$, $\quad 3x > 12$
$\quad x > 4$

If $3x < 0$, $\quad -3x > 12$
$\quad x < -4$

The solution to $|3x| > 12$ is $x > 4$ or $x < -4$.

(b) $|4b-6| \leq 18$

$$\begin{array}{lrcl} \text{If} \quad 4b-6 \geq 0, & 4b-6 & \leq & 18 \\ & 4b & \leq & 24 \\ & b & \leq & 6 \end{array}$$

For this case we want the values of b that satisfy both $4b-6 \geq 0$ and $b \leq 6$. This means $b \geq \frac{3}{2}$ and $b \leq 6$, so we want $\frac{3}{2} \leq b \leq 6$.

$$\begin{array}{lrcl} \text{If} \quad 4b-6 < 0, & -(4b-6) & \leq & 18 \\ & 6-4b & \leq & 18 \\ & -4b & \leq & 12 \\ & b & \geq & -3 \end{array}$$

For this case we want the values of b that satisfy both $4b-6 < 0$ and $b \geq -3$. This means $b < \frac{3}{2}$ and $b \geq -3$, so we want $-3 \leq b < \frac{3}{2}$.

The solution to $|4b-6| \leq 18$ is $\frac{3}{2} \leq b \leq 6$ and $-3 \leq b < \frac{3}{2}$, which is $-3 \leq b \leq 6$.

(c) $7 \geq |x+14|$

$$\begin{array}{lrcl} \text{If} \quad x+14 \geq 0, & 7 & \geq & x+14 \\ & -x & \geq & 7 \\ & x & \leq & -7 \end{array}$$

For this case we want the values of x that satisfy both $x+14 \geq 0$ and $x \leq -7$. This means $x \geq -14$ and $x \leq -7$, so we want $-14 \leq x \leq -7$.

$$\begin{array}{lrcl} \text{If} \quad x+14 < 0, & 7 & \geq & -x-14 \\ & 7+x & \geq & -14 \\ & x & \geq & -21 \end{array}$$

For this case we want the values of x that satisfy both $x+14 < 0$ and $x \geq -21$. This means $x < -14$ and $x \geq -21$, so we want $-21 \leq x < -14$.

The solution to $7 \geq |x+14|$ is $-14 \leq x \leq -7$ and $-21 \leq x < -14$, which is $-21 \leq x \leq -7$.

(d) $9 < |3c-6|$

$$\begin{array}{lrcl} \text{If} \quad 3c-6 \geq 0, & 9 & < & 3c-6 \\ & -3c & < & -15 \\ & c & > & 5 \end{array}$$

For this case we want the values of c that satisfy both $3c-6 \geq 0$ and $c > 5$. This means $c \geq 2$ and $c > 5$, so we want $c > 5$.

$$\begin{array}{lrcl} \text{If} \quad 3c-6 < 0, & 9 & < & -(3c-6) \\ & 9 & < & 6-3c \\ & 3c & < & -3 \\ & c & < & -1 \end{array}$$

For this case we want the values of c that satisfy both $3c-6 < 0$ and $c < -1$. This means $c < 2$ and $c < -1$, so we want $c < -1$.

The solution to $9 < |3c-6|$ is $c > 5$ or $c < -1$.

11.8 Sigma notation

A8.1 **(a)** $\sum_{n=1}^{3} 2n$ is the sum for $n = 1, 2, 3$ of $2n$. This is $2 \times 1 + 2 \times 2 + 2 \times 3$, which is equal to $2 + 4 + 6$.

(b) $\sum_{n=-1}^{2} 3^n$ is the sum for $n = -1, 0, 1, 2$ of 3^n. This is $3^{-1} + 3^0 + 3^1 + 3^2$, which is equal to $\frac{1}{3} + 1 + 3 + 9$.

(c) $\sum_{i=0}^{5} i(i+2)$ is the sum from $i = 0$ to 5 of $i(i+2)$. So this is simply $0(0+2) + 1(1+2) + 2(2+2) + 3(3+2) + 4(4+2) + 5(5+2)$, which is equal to $0 + 2 + 8 + 15 + 24 + 35$.

A8.2 **(a)** The sum $3 + 4 + 5 + 6 + 7 + 8$ is the sum of all integers from 3 to 8. In sigma notation this is $\sum_{i=3}^{8} i$.

(b) The sum $-2+1+4+7$ is the sum of four terms, each of which represents an increase of 3 over the previous term. So if we consider the first term to be $0\times3-2$ (using 3 because this is the difference between consecutive terms), we can use sigma notation to get $\sum_{i=0}^{3} 3i - 2$.

(c) The sum $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{100}$ is the sum of 100 terms, each of which is a fraction where the numerator is 1 and the denominator is the number between 1 and 100 which corresponds to the particular term. In sigma notation this is $\sum_{i=1}^{100} \frac{1}{i}$.

A8.3 $\sum_{j=3}^{6} (-1)^j\,(2j+n^{j-2})$ corresponds to the sum with $j = 3, 4, 5, 6$ of $(-1)^j\,(2j+n^{j-2})$. This is $(-1)^3\,(2\times3+n^{3-2}) + (-1)^4\,(2\times4+n^{4-2}) + (-1)^5\,(2\times5+n^{5-2}) + (-1)^6\,(2\times 6 + n^{6-2})$ which is equivalent to $-(6+n) + (8+n^2) - (10+n^3) + (12+n^4)$.

A8.4 The sum $3x_1 - 5x_2 + 7x_3 - 9x_4$ is a sum with alternating plus and minus signs for each term, a factor out the front that increases by 2 each time, and a power of x that increases by 1 for each consecutive term. If we let the variable over which the sum is to be performed be i, and allow this to go from 1 to 4, the first term could be $(2i+1)x_i$. This gives the right terms, but now the positives and negatives must be dealt with. Since $(-1)^0 = 1$ and $(-1)^1 = -1$, we can include the term $(-1)^{i-1}$ to give the full sum. This gives a final answer of $\sum_{i=1}^{4} (-1)^{i-1}\,(2i+1)x_i$.

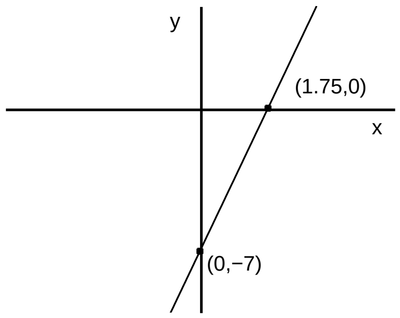
11.9 Graphing lines and parabolas

A9.1 **(a)** $y = 4x - 7$

x-intercept: $0 = 4x - 7$, so $x = \frac{7}{4}$

y-intercept: $y = 0 - 7$, so $y = -7$

Points to plot: $(1.75, 0)$ and $(0, -7)$.

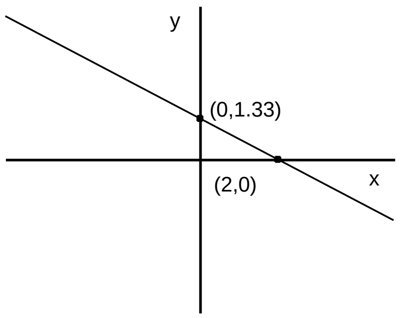


(b) $3y = -2x + 4$

x-intercept: $0 = -2x + 4$, so $x = 2$

y-intercept: $3y = 0 + 4$, so $y = \frac{4}{3}$

Points to plot: $(2, 0)$ and $(0, 1.33)$.

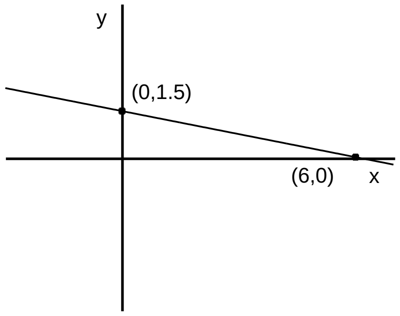


(c) $-2y = \frac{x}{2} - 3$

x-intercept: $0 = \frac{x}{2} - 3$, so $x = 6$

y-intercept: $-2y = 0 - 3$, so $y = \frac{3}{2}$

Points to plot: $(6, 0)$ and $(0, 1.5)$.



A9.2 **(a)** The two points given are $(1, 6)$ and $(2, 16)$.
The gradient, m:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 6}{2 - 1} = 10$$

The y-intercept, c:
Using the point (1,6) on the line gives

$$\begin{aligned} y &= mx + c \\ 6 &= 10 \times 1 + c \\ 6 &= 10 + c \\ -4 &= c \end{aligned}$$

Substituting m and c into the general equation we now have the equation for the line given in the graph above, $y = 10x - 4$.

(b) The two points given are $(2, 0)$ and $(4, -6)$.
The gradient, m:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{4 - 2} = \frac{-6}{2} = -3$$

The y-intercept, c:
Using the point (2,0) on the line gives

$$\begin{aligned} y &= mx + c \\ 0 &= -3 \times 2 + c \\ 0 &= -6 + c \\ 6 &= c \end{aligned}$$

Substituting m and c into the general equation we now have the equation for the line given in the graph above, $y = -3x + 6$.

(c) The two points given are $(4, -4)$ and $(8, -7)$.
The gradient, m:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - -4}{8 - 4} = \frac{-3}{4}$$

The y-intercept, c:
Using the point $(4, -4)$ on the line gives

$$\begin{aligned} y &= mx + c \\ -4 &= -\frac{3}{4} \times 4 + c \\ -4 &= -3 + c \\ -1 &= c \end{aligned}$$

Substituting m and c into the general equation we now have the equation for the line given in the graph above, $y = -\frac{3}{4}x - 1$.

A9.3 **(a)** $y = 4x^2 - 25$

x-intercept: $0 = 4x^2 - 25$, so $0 = (2x - 5)(2x + 5)$,

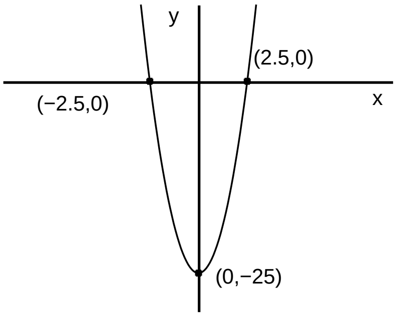
$x = \frac{5}{2}$ and $x = -\frac{5}{2}$

y-intercept: $y = 0 - 25$, so $y = -25$

Turning point: $x = \frac{-b}{2a} = \frac{0}{8} = 0$

$y = 4(0)^2 - 25 = -25$

Points to plot: $(-2.5, 0)$, $(2.5, 0)$ and $(0, -25)$.



(b) $y = x^2 + 4x + 5$

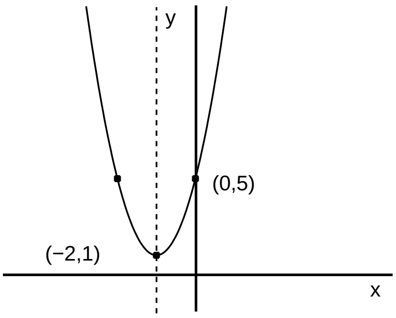
x-intercept: $0 = x^2 + 4x + 5$, using the quadratic formula there are no real roots.

y-intercept: $y = 0 + 4(0) + 5$, so $y = 5$

Turning point: $x = \frac{-b}{2a} = \frac{-4}{2} = -2$

$y = (-2)^2 + 4(-2) + 5 = 1$

Points to plot: $(0, 5)$ and $(-2, 1)$.



(c) $y = -x^2 + 4x - 4$

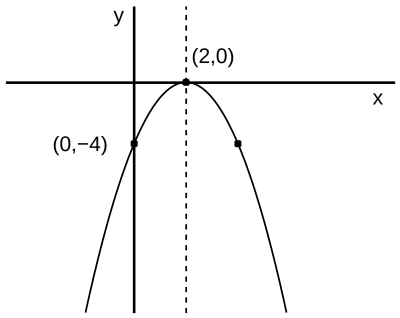
x-intercept: $0 = -x^2 + 4x - 4$, so $0 = (-x + 2)(x - 2)$, so $x = 2$

y-intercept: $y = 0 - 4(0) - 4$, $y = -4$

Turning point: $x = \frac{-b}{2a} = \frac{-4}{-2} = 2$

$y = -(2)^2 + 4(2) - 4 = 0$

Points to plot: $(2, 0)$ and $(0, -4)$.


y
(2,0)
x
(0,−4)

11.10 Derivatives and anti-derivatives

A10.1 **(a)**
$$\begin{aligned} f(x) &= 4x^2 + 2x + 6 \\ f'(x) &= 2 \times 4x^{(2-1)} + 2x^{(1-1)} + 0 \\ f'(x) &= 8x + 2 \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 9x^3 - 5x^{-4} + 5 \\ f'(x) &= 3 \times 9x^{(3-1)} - -4 \times 5x^{(-4-1)} + 0 \\ f'(x) &= 27x^2 + 20x^{-5} \end{aligned}$$

(c)
$$\begin{aligned} f(x) &= \sqrt{x} + 2x^2 \\ f'(x) &= \frac{1}{2} \times x^{(\frac{1}{2}-1)} + 2 \times 2x^{(2-1)} \\ f'(x) &= \frac{1}{2\sqrt{x}} + 4x \end{aligned}$$

(d)
$$\begin{aligned} f(a) &= a^4 - a^2 + a \\ f'(a) &= 4 \times a^{(4-1)} - 2 \times a^{(2-1)} + a^{(1-1)} \\ f'(a) &= 4a^3 - 2a + 1 \end{aligned}$$

A10.2 **(a)**
$$\begin{aligned} f(x) &= 4\cos x \\ f'(x) &= 4 \times -\sin x \\ f'(x) &= -4\sin x \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 2\sin x + 8x \\ f'(x) &= 2 \times \cos x + 8x^{(1-1)} \\ f'(x) &= 2\cos x + 8 \end{aligned}$$

(c)
$$\begin{aligned} f(a) &= a^2 + \sin a - 4 \\ f'(a) &= 2 \times a^{(2-1)} + \cos a + 0 \\ f'(a) &= 2a + \cos a \end{aligned}$$

(d)
$$\begin{aligned} f(x) &= x^3 - 3\cos x + 2x \\ f'(x) &= 3 \times x^{(3-1)} - 3 \times -\sin x + 2x^{(1-1)} \\ f'(x) &= 3x^2 + 3\sin x + 2 \end{aligned}$$

A10.3 **(a)**
$$\begin{aligned} f(x) &= 5e^x \\ f'(x) &= 5e^x \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= -7e^x + x^2 \\ f'(x) &= -7e^x + 2 \times x^{(2-1)} \\ f'(x) &= -7e^x + 2x \end{aligned}$$

(c)
$$\begin{aligned} f(x) &= 2\ln(x) \\ f'(x) &= 2 \times \frac{1}{x} \\ f'(x) &= \frac{2}{x} \end{aligned}$$

(d)
$$\begin{aligned} f(a) &= 4a^2 + \ln(a) \\ f'(a) &= 2 \times 4a^{(2-1)} + \frac{1}{a} \\ f'(a) &= 8a + \frac{1}{a} \end{aligned}$$