Algebraic Reminders

Factoring

Common factors $ax + ay = a(x + y)$

- $5x - 10y = 5(x - 2y)$ common factor 5
- $2(k+1) + (k+1)(k-4) = (k+1)[2 + (k-4)] = (k+1)(k-2)$
  common factor $(k+1)$
- $3^{x+1} - 3^x = 3^x(3-1) = 2 \cdot 3^x$ common factor 3^x since $3^{x+1} = 3^x \cdot 3$

Multiplying binomials $(x+a)(x+b) = x^2 + bx + ax + ab = x^2 + (a+b)x + ab$

- $(x+2)(3x-5) = 3x^2 - 5x + 6x - 10 = 3x^2 + x - 10$
- $$\begin{aligned}(y-1)(y+4)(y+5) &= (y^2 + 3y - 4)(y+5) \\ &= y^3 + 5y^2 + 3y^2 + 15y - 4y - 20 \\ &= y^3 + 8y^2 + 11y - 20\end{aligned}$$

Trinomials $x^2 + (a+b)x + ab = (x+a)(x+b)$
(If the sign of the constant term is positive, then you want either two positive or two negative integers for a and b. If the sign of the contant term is negative, then you want one positive and one negative integer for a and b.)

- $x^2 + 5x + 6 = (x+2)(x+3)$
  two positive integers whose sum is 5 and whose product is 6
- $y^2 - y - 6 = (y+2)(y-3)$
  two integers (positive, negative) whose sum is -1 and whose product is -6:
- $k^2 - 7k + 12 = (k-3)(k-4)$
  two negative integers whose sum is -7 and whose product is 12

Perfect square trinomials $x^2 + 2ax + a^2 = (x+a)^2$ $x^2 - 2ax + a^2 = (x-a)^2$
(this is just factoring a trinomial in which the integers a and b are equal)

- $k^2 + 2k + 1 = (k+1)^2$ here $a = 1$
- $x^2 - 4x + 4 = (x-2)^2$ here $a = 2$

Difference of squares $m^2 - n^2 = (m-n)(m+n)$

- $x^2 - 9 = (x-3)(x+3)$
- $25a^4 - 16b^2 = (5a^2 - 4b)(5a^2 + 4b)$

Fractions

Adding or subtracting To add or subtract two fractions you need to write each fraction with a common denominator. Then add or subtract the numerators and put this over the common denominator.

- $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$
- $$\begin{aligned} \frac{2}{k+1} - \frac{k+1}{k+2} &= \frac{2(k+2)}{(k+1)(k+2)} - \frac{(k+1)^2}{(k+1)(k+2)} \\ &= \frac{2k+4-(k^2+2k+1)}{(k+1)(k+2)} \\ &= \frac{-k^2+3}{(k+1)(k+2)} \end{aligned}$$

Multiplying To multiply fractions, multiply the numerators and multiply the denominators.

- $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- $\frac{4(k+1)}{k} \cdot \frac{2k^2}{(k+1)(k+2)} = \frac{8k^2(k+1)}{k(k+1)(k+2)} = \frac{8k}{k+2}$

Dividing To divide fractions, multiply the first fraction by the reciprocal of the second fraction (the fraction flipped upside down).

- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Laws of Exponents

- $x^0 = 1$
- $x^{-a} = \frac{1}{x^a}$
- $x^a \cdot x^b = x^{a+b}$
- $x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$
- $(x^a)^b = x^{ab}$
- $(xy)^a = x^a \cdot y^a$