1. Section 1.1 questions from textbook

10b. $p \wedge \sim q$ **10d.** $\sim p \wedge q \wedge \sim r$ **10e.** $\sim p \vee (q \wedge r)$

22. Since the entries in the final two columns of the truth table are identical, the statements $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent.

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

32. Let t be statement "the train is late" and let w be the statement "my watch is fast". Then the statement "The train is late or my watch is fast" can be written as $t \vee w$. By De Morgan's laws

$$\sim (t \vee w) \equiv \sim t \wedge \sim w,$$

so the negation of the given statement is "The train is not late and my watch is not fast."

40. The final column of the truth table shows that the statement form $(\sim p \vee q) \vee (p \wedge \sim q)$ is a tautology.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

Section 1.2 questions from textbook.

11. The truth values of the statement form $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$ are given in the truth table below.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$(p \to (q \to r) \leftrightarrow ((p \wedge q) \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

16g. Let p represent the statement "n is divisible by 6", let q represent the statement "n is divisible by 2", and let r represent the statement "n is divisible by 3". Then the given statement form can be written as $p \to (q \wedge r)$. The negation of this is

$$\sim (p \to (q \wedge r)) \equiv \sim (\sim p \vee (q \wedge r)) \equiv p \wedge \sim (q \wedge r) \equiv p \wedge (\sim q \vee \sim r).$$

In words this is "n is divisible by 6 but n is neither divisible by 2 nor divisible by 3".

2. There are 5 mistakes in the following table. Circle each of them.

p	q	$p \vee q$	$p \to q$	$p \leftrightarrow q$	$p \wedge q$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	F	F	F	F	T

3. For each argument, the solution is given using both methods.

(i) Let m represent the statement "Bruce is studying for a mathematics degree", let e represent the statement "Bruce is studying for an economics degree", and let p represent the statement "Bruce is required to pass MATH1061".

The the premises can be written as $(m \vee e)$ and $(m \to p)$. The conclusion can be written as $(e \vee p)$. The argument can be written as

$$[(m \vee e) \wedge (m \to p)] \to (e \vee p).$$

To determine whether or not this is a valid argument, we can use the following truth table.

m	e	p	$m \vee e$	$m \to p$	$e \vee p$	critical row
T	T	T	T	T	T	*
T	T	F	T	F	T	
T	F	T	T	T	T	*
T	F	F	T	F	F	
F	T	T	T	T	T	*
F	T	F	T	T	T	*
F	F	T	F	T	T	
F	F	F	F	T	F	

Since, in each of the critical rows, the conclusion is true, this argument is valid.

Alternatively, we could use the following method. If the argument were invalid, then there must be truth values for m, e and p such that the premises are true and the conclusion is false.

For the conclusion $e \vee p$ to be false, we need e to be False and p to be False.

For the premise $m \to p$ to be true, we need either m to be false or both m and p to be true. But since we know that we need p to be false, we must have m as False.

But now it is impossible for the premise $m \vee e$ to be true.

Hence it is impossible to find truth values that make this argument invalid, so the argument must be valid.

(ii) Let k represent the statement "I eat Kentucky Fried Chicken", let b represent the statement "I am beautiful", and let h represent the statement "I am happy".

The the premises can be written as $(k \to b)$, $(b \to h)$ and h. The conclusion can be written as k. The argument can be written as

$$[(k \to b) \wedge (b \to h) \wedge h] \to k.$$

To determine whether or not this is a valid argument, we can use the following truth table.

k	b	h	$k \to b$	$b \to h$	h	k	critical row
T	T	T	T	T	T	T	*
T	T	F	T	F	F	T	
T	F	T	F	T	T	T	
T	F	F	F	T	F	T	
F	T	T	T	T	T	F	*
F	T	F	T	F	F	F	
F	F	T	T	T	T	F	*
F	F	F	T	T	F	F	

Since, there is a critical row in which all of the premises are true but the conclusion is false, this argument is invalid.

Alternatively, we could use the following method. If the argument were invalid, then there must be truth values for k, b and h such that the premises are true and the conclusion is false.

For the conclusion k to be false, we need k to be False.

For the premise h to be true, we need h to be True.

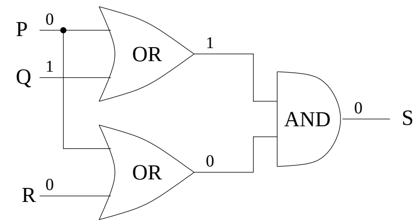
For the premise $k \to b$ to be true, with k false, we can have b either true or false.

For the premise $b \to h$ to be true, with h true, we can have b either true or false.

Hence k FALSE, b TRUE, h TRUE are truth values that make the premises true but the conclusion false. Therefore this argument is invalid.

4. Section 1.4 questions from textbook.

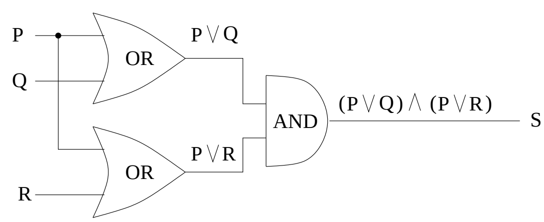
4. With input signals $P = 0$, $Q = 1$ and $R = 0$, the output is $S = 0$.



8. The corresponding input/output table is

P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

12. The corresponding boolean expression is $(P \vee Q) \wedge (P \vee R)$. Note that this expression is logically equivalent to $P \vee (Q \wedge R)$.



5. Label the copies of the letter X as X_1, X_2, X_3 and X_4 from left to right.

(i) The four input/output tables are shown below.

a	b	c	X_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

a	b	c	X_2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

a	b	c	X_3
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

a	b	c	X_4
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

(ii) A Boolean expression for the light X_1 is

$$(a \wedge b \wedge c) \vee (a \wedge b \wedge \sim c) \vee (a \wedge \sim b \wedge \sim c).$$

You can show, using a truth table, that this expression is logically equivalent to

$$a \wedge (b \vee \sim c).$$

Now the light X_2 lights up whenever X_1 lights up and also when $a = 0$ and $b = c = 1$, so a Boolean expression for X_2 is

$$[a \wedge (b \vee \sim c)] \vee (\sim a \wedge b \wedge c).$$

This is logically equivalent to $(b \wedge c) \vee (a \wedge \sim c)$.

A Boolean expression for X_3 is

$$[a \wedge (b \vee \sim c)] \vee (\sim a \wedge b \wedge c) \vee (\sim a \wedge b \wedge \sim c).$$

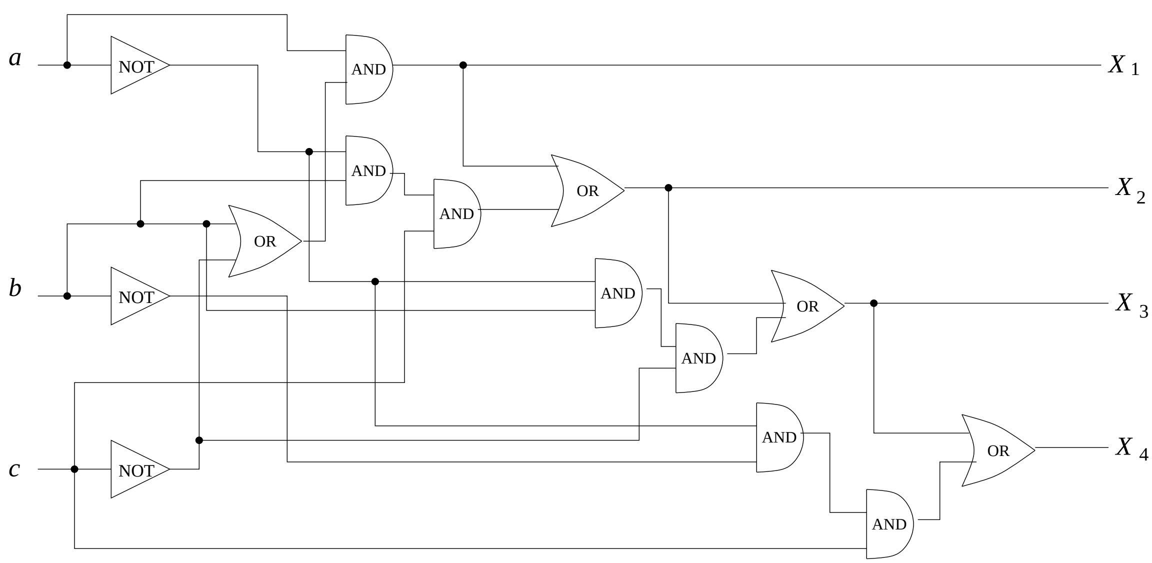
This is logically equivalent to $[a \wedge (b \vee \sim c)] \vee (\sim a \wedge b)$.

A Boolean expression for X_4 is

$$[a \wedge (b \vee \sim c)] \vee (\sim a \wedge b \wedge c) \vee (\sim a \wedge b \wedge \sim c) \vee (\sim a \wedge \sim b \wedge c).$$

This is logically equivalent to $[a \wedge (b \vee \sim c)] \vee [\sim a \wedge (b \vee c)]$.

There are many different ways to construct the circuit. One possibility is



6. Section 2.1 questions from textbook.

7. There are many answers. Choose $x = 9$ and $y = 16$. Then $\sqrt{x+y} = \sqrt{25} = 5$ and $\sqrt{x} + \sqrt{y} = \sqrt{9} + \sqrt{16} = 3 + 4 = 7$. Therefore, $\exists$ real numbers x and y such that $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.

11b. $\forall$ real numbers x, x is positive, negative or zero.
Alternatively, $\forall$ real numbers x, $(x > 0) \vee (x < 0) \vee (x = 0)$.

12b. $\exists$ a real number x such that x is rational.
Alternatively, $\exists$ a real number x such that $x \in \mathbb{Q}$.

28b. This statement is true. To see this, check all values of $x \in D$ that are less than 0. The values $-48, -14$, and -8 are all even.

28e. This statement is false. The number 36 is in the set D. Its ones digit is 6 but its tens digit is not 1 or 2.

7. (i) For every real number x, there exists a real number y such that x is greater than or equal to y.

This statement is true.

The negation of this statement is $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $x < y$.

(ii) There exists an integer z such that for every real number x and for every real number y, $x + y \geq z$.

This statement is false.

The negation of this statement is $\forall z \in \mathbb{Z}, \exists x \in \mathbb{R}$ and $\exists y \in \mathbb{R}$ such that $x + y < z$.