Complete all of the following problems and hand in your solutions to your tutor by 1:50pm on Tuesday 26 August, 2003. **Make sure that your name and student number are on each sheet of your answers**. Solutions to all the problems will be distributed later.

Completing 5 out of the 6 assignments is compulsory and each of the five assignments will contribute 4% towards your final grade. Late assignments will not be accepted unless you have a very good excuse.

1. Complete the following questions from the textbook:

- Section 3.1, pages 124-125: Questions 7, 26, 41.
- Section 3.2, pages 130-131: Question 15.
- Section 3.3, pages 138-139: Questions 8, 15, 22, 31b.
- Section 3.4, pages 146-147: Questions 6, 10b, 23a.
- Section 3.5, pages 153-154: Questions 10b, 13a.
- Section 3.6, page 161: Questions 8, 22.

2. Each of the following "proofs" is incorrect. Explain why each "proof" is wrong.

(a) Any product of four consecutive integers is one less than a perfect square.

Proof: Consider the four consecutive integers 3, 4, 5, 6. The product

$$3 \cdot 4 \cdot 5 \cdot 6 = 360.$$

Now $361 = (19)^2$ so 360 is one less than a perfect square. Hence any product of four consecutive integers is one less than a perfect square.

(b) For all integers a, b, c, if $a \mid bc$, then $a \mid b$.

Proof: Let a, b, c be integers and suppose that $a \mid b$. Then $b = ra$ for some integer r. Multiplying both sides of this equation by c we get

$$bc = (ra)c = (rc)a.$$

Since rc is an integer, we know that $a \mid bc$. Hence if $a \mid bc$, then $a \mid b$.

(c) The difference of any two odd integers is even.

Proof: Let m and n be odd integers. Suppose that $m - n$ is even. Thus $m - n = 2r$ for some integer r. Now since m and n are both odd, we have

$$m = 2s + 1 \text{ for some integer } s, \quad \text{and} \quad n = 2t + 1 \text{ for some integer } t.$$

Thus

$$m - n = (2s + 1) - (2t + 1) = 2r,$$

so the difference of any two odd integers is even.

3. Use the Euclidean Algorithm to calculate the greatest common divisor of each of the following pairs of numbers.

(a) 49, 63

(b) 238, 14

(c) 1550, 250

4. Find a solution to the linear Diophantine equation $1550c + 250d = 46500$.

5. Show that there does not exist a point with integer co-ordinates that lies on the line $63x + 49y = 4$.

6. (Bonus Question)
A company has \$46,500.00 to spend on new computers and desks. Each computer costs \$1550.00 and each desk costs \$250.00. How many computers and desks should the company buy in order to spend *all* the money, while ensuring that they get approximately the same number of desks as computers?