Complete all of the following problems and hand in your solutions to your tutor by 1:50pm on Tuesday 8 September, 2003. **Make sure that your name and student number are on each sheet of your answers**. Solutions to all the problems will be distributed later.

Completing 5 out of the 6 assignments is compulsory and each of the five assignments will contribute 4% towards your final grade. Late assignments will not be accepted unless you have a very good excuse.

1. Complete the following problems from the textbook:

- Section 4.1, pages 192-193: Questions 21, 30, 34, 49
- Section 4.2, pages 204-205: Question 15 (Hints: You are asked to prove that $\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\frac{n+1}{2n}$. You can use the fact that $\left(1-\frac{1}{(k+1)^2}\right)=\frac{k(k+2)}{(k+1)^2}$.
- Section 4.3, pages 210-211: Questions 20b, 24

2. There are n people attending a dinner party and as people arrive they shake hands. Each person shakes hands exactly once with each other person.

(a) Explain (in english) why the total number of handshakes is given by

$$1+2+3+\ldots+(n-3)+(n-2)+(n-1).$$

(b) Write the expression from part (a) in summation notation.

(c) Use mathematical induction to prove that, provided that there are at least two people at the party, the number of handshakes is

$$\frac{n(n-1)}{2}.$$

(d) The formula from part (c) appears surprisingly often in real-life. Suppose that there are n teams in a rugby league competition. Every team A plays every other team B twice, once at the home ground for team A, and the other time at the home ground for team B. Using the result from part (c), explain (in english) why, in a competition with n teams, there will be $n(n-1)$ games played in a whole season. How many matches would be played with 14 teams?

3. Complete the following problems from the textbook:

- Section 5.1, pages 242–243: Questions 6cd, 9ab, 13c, 15de
- Section 5.3: pages 266–268: Questions 38, 41b

4. Use propositions and a truth table to prove that for arbitrary sets A and B,

$$\text{if } A \subseteq B, \text{ then } (A \cap B) \subseteq B.$$

5. Use Venn diagrams to illustrate that for arbitrary sets A, B and C,

$$(A - B) \cap (C - B) = (A \cap C) - B.$$

6. Let A be the set containing the empty set and $B = \{A\}$. State whether each of the following statements is true or false.

- The empty set is a member of A.
- The empty set is a member of B.
- The empty set is a subset of A.
- The empty set is a subset of B.
- A is a subset of B.
- A is a member of B.