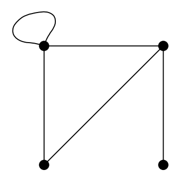
1. Section 11.1 questions from textbook

9. **(i)** The edges incident on v_1 are: e_1, e_2 and e_7.
(ii) The vertices adjacent to v_3 are: v_1 and v_2.
(iii) The edges adjacent to e_1 are: e_2 and e_7.
(iv) The loops are: e_1 and e_3.
(v) The parallel edges are: e_4 and e_5.
(vi) The isolated vertex is: v_4.
(vii) The degree of v_3 is: 2.
(viii) The total degree of the graph is: $4+6+2+0+2=14$.

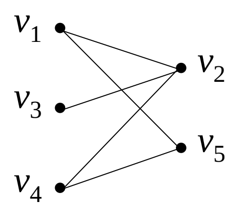
17. The total degree of such a graph would be $1+1+1+4=7$ which is odd. Therefore, no such graph exists.

18. Such a graph is shown.



37. **(d)** This graph is not bipartite because there is a triangle on the vertices v_2, v_4, v_5. Thus, v_2 and v_4 must be in different parts and then v_5 cannot be in either part.

(e) This graph is bipartite. It has been redrawn below.



Section 11.2 questions from textbook.

2. **(a)** $v_1e_2v_2e_3v_3e_4v_4e_5v_2e_2v_1e_1v_0$ is only a walk.
(b) $v_2v_3v_4v_5v_2$ is a path, a closed walk, a circuit and a simple circuit.
(d) $v_2v_1v_5v_2v_3v_4v_2$ is a path, a closed walk and a circuit.

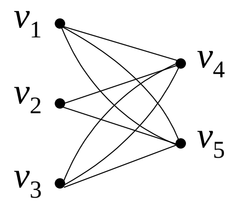
15. Every vertex in this graph has even degree, so this graph has an Euler circuit. Once such Euler circuit is:

$$t\ s\ r\ z\ s\ u\ z\ y\ u\ w\ y\ x\ w\ v\ u\ t.$$

16. This graph is disconnected, so it does not have an Euler circuit.

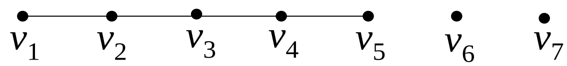
Section 11.3 questions from textbook.

22a. A graph with this adjacency matrix is shown below. This graph is bipartite.



Section 11.5 questions from textbook.

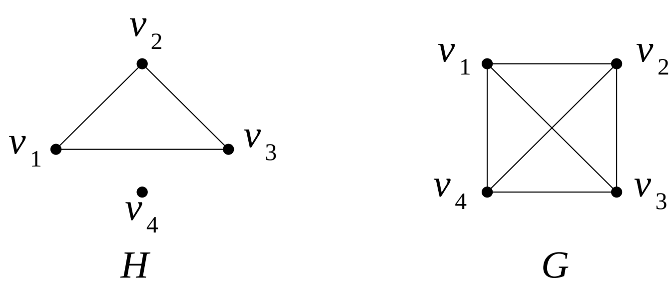
15. A circuit-free graph with seven vertices and four edges is shown.



16. A tree with twelve vertices would have eleven edges.
Hence, a tree with twelve vertices and 15 edges does not exist.

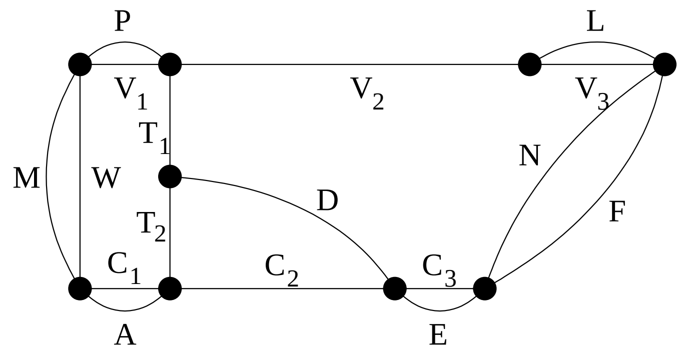
18. A tree with five vertices would have four edges. A graph that has total degree 10 has $10 \div 2 = 5$ edges. Hence, a tree with five vertices and total degree 10 does not exist.

2. **(a)** The graph H is a subgraph of the graph G. The following vertex labelling of H and G shows that each vertex and each edge of H are also in G.



(b) The graph H is not a subgraph of the graph G. It is not possible to find four edges that form a triangle with a loop in the graph G.

3. The following graph represents the postman's route. The vertices represent the points at which he has a choice of road, and the edges represent the roads. The edges have been labelled with the first letter of the corresponding road name, with sections of the same road labelled by subscripts.



(a) If the postman could deliver the mail along his route by walking along each road exactly once and finish where he started, then the above graph would have an Euler circuit. But the graph has two vertices of odd degree, so it does not have an Euler circuit. Hence the postman cannot deliver his mail by walking along each road exactly once and finish where he started.

(b) The postman should follow an Euler path in the above graph. He should be dropped off at the corner of Disc Drive and Toad Road and be picked up at the T-junction of Luminescent Crescent and Vindaloo Avenue. Once possible path he could follow is:

- Start at the corner of Disc Drive and Toad Road.
- Walk along Disc Drive, Effervescent Crescent, No Way, Luminescent Crescent.
- Turn right onto Vindaloo Avenue and walk east.
- Walk south on Ferris Terrace and then west on Cheat Street.
- Turn left onto Toad Road and walk north to the end of that street.
- Walk along Pheasant Crescent, Wart Court, Adolescent Crescent.
- Turn left and walk west on Cheat Street, then north on Morose Close.
- Walk east on Vindaloo Avenue until reaching the pickup point.

This Euler path can be written as:

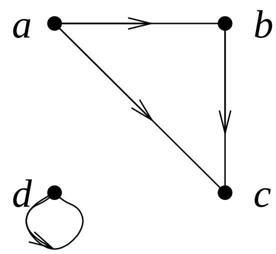
$$D\ E\ N\ L\ V_3\ F\ C_3\ C_2\ T_2\ T_1\ P\ W\ A\ C_1\ M\ V_1\ V_2$$

4. Section 10.1 questions from textbook.

2. (a) No. 2 is not related to 4 by S since $2 \not\geq 4$.
Yes. 4 is related to 3 by S.
Yes. $(4,4) \in S$.
No. $(3,2) \notin S$ because 2 is not in the set D.

(b) $S = \{(3,3),(4,3),(5,3),(4,4),(5,4)\}$.

24. The directed graph representing the binary relation S on B is as follows.



Section 10.2 questions from textbook.

13. The relation C is not reflexive, since it is not true that $\forall x \in \mathbb{R},\ x\ C\ x$. For example, 2 is a real number, but $2^2 + 2^2 \neq 1$.

The relation C is symmetric, since it is true that $\forall x, y \in \mathbb{R}$, if $x\ C\ y$, then $y\ C\ x$. Suppose that $x, y \in \mathbb{R}$ and $x\ C\ y$. Then $x^2 + y^2 = 1$, and so $y^2 + x^2 = 1$. Therefore $y\ C\ x$.

The relation C is not transitive, since it is not true that $\forall x, y, z \in \mathbb{R}$, if $x\ C\ y$ and $y\ C\ z$, then $x\ C\ z$. Suppose that

$$x = \frac{\sqrt{3}}{2}, \quad y = \frac{1}{2}, \quad z = -\frac{\sqrt{3}}{2}.$$

Then $x^2 + y^2 = \frac{3}{4} + \frac{1}{4} = 1$ and and $y^2 + z^2 = \frac{1}{4} + \frac{3}{4} = 1$, but $x^2 + z^2 = \frac{3}{4} + \frac{3}{4} \neq 1$.

24. The relation $\mathcal{R}$ is not reflexive, since it is not true that $\forall A \in \mathcal{P}(X), A\ \mathcal{R}\ A$. For example, consider $A = \{a, b\}$, then $A \in \mathcal{P}(X)$, $n(A) = 2$ and $2 \not< 2$.

The relation C is not symmetric, since it is not true that $\forall A, B \in \mathcal{P}(X)$, if $A\ \mathcal{R}\ B$, then $B\ \mathcal{R}\ A$. Suppose that $A, B \in \mathcal{P}(X)$ and $A\ \mathcal{R}\ B$. Then $n(A) < n(B)$, so $n(B) \not< n(A)$. Therefore B is not related to A by $\mathcal{R}$.

The relation C is transitive, since it is true that $\forall A, B, C \in \mathcal{P}(X)$, if $A\ \mathcal{R}\ B$ and $B\ \mathcal{R}\ C$, then $A\ \mathcal{R}\ C$. Suppose that $A, B, C \in \mathcal{P}(X)$ and $A\ \mathcal{R}\ B$ and $B\ \mathcal{R}\ C$. Then $n(A) < n(B)$ and $n(B) < n(C)$. Thus $n(A) < n(C)$, so $A\ \mathcal{R}\ C$.

Section 10.3 questions from textbook.

3. There are 3 equivalence classes of R. They are:

$$[a] = \{a\}, \quad [c] = \{c\} \quad \text{and} \quad [b] = [d] = \{b, d\}.$$

Note that the questions on Section 10.5 have been moved to Assignment 5.

5. (a) The relation S is relexive, since it is true that $\forall a \in \mathbb{Z},\ a\ S\ a$.
If $a \in \mathbb{Z}$ then $4 \mid (a-a)$ so $a \equiv a \pmod 4$.

The relation S is symmetric, since it is true that

$$\forall a, b \in \mathbb{Z},\ \text{if } a\ S\ b,\ \text{then } b\ S\ a.$$

Suppose that $a, b \in \mathbb{Z}$ and $a\ S\ b$. Then $a \equiv b \pmod 4$ so $4 \mid (a-b)$. Hence, $a-b = 4k$ for some integer k and so $b-a = 4(-k)$. Thus $4 \mid (b-a)$ so $b \equiv a \pmod 4$.

The relation S is transitive, since it is true that

$$\forall a, b, c \in \mathbb{Z},\ \text{if } a\ S\ b \text{ and } b\ S\ c,\ \text{then } a\ S\ c.$$

Suppose that $a, b, c \in \mathbb{Z}$ and $a\ S\ b$ and $b\ S\ c$. Then $a \equiv b \pmod 4$ and $b \equiv c \pmod 4$, so $4 \mid (a-b)$ and $4 \mid (b-c)$. Thus $a-b = 4k$ and $b-c = 4h$ for some integers k and h. Thus $a-c = (a-b)+(b-c) = 4k+4h = 4(k+h)$. Hence $4 \mid (a-c)$ and so $a\ S\ c$.

Since S is reflexive, symmetric and transitive, it is an equivalence relation.

(b) There are four equivalence classes of S. They correspond to the four possible remainders 0, 1, 2, 3 resulting from division by 4.

$[0]_S = \{x \in \mathbb{Z} \mid x = 4k,\ \text{for some integer } k\}$
$[1]_S = \{x \in \mathbb{Z} \mid x = 4k+1,\ \text{for some integer } k\}$
$[2]_S = \{x \in \mathbb{Z} \mid x = 4k+2,\ \text{for some integer } k\}$
$[3]_S = \{x \in \mathbb{Z} \mid x = 4k+3,\ \text{for some integer } k\}$