Complete all of the following problems and hand in your solutions to the lecturer by 10:00am on Friday 17 October, 2003. **Make sure that your name and student number are on each sheet of your answers**. Solutions to all the problems will be distributed later.

Completing 5 out of the 6 assignments is compulsory and each of the five assignments will contribute 4% towards your final grade. Late assignments will not be accepted unless you have a very good excuse.

1. Complete the following problems from the textbook:

- Section 10.5: pages 599–601: Questions 1cd, 7
- Section 7.1, pages 354-357: Questions 2, 3cd, 12bd, 29
- Section 7.3, pages 384-387: Questions 4, 5bc, 9, 36
- Section 7.4, pages 399-400: Questions 6, 13, 28
- Section 7.5, pages 410-411: Questions 7, 9, 21

2. At one of the parking lots in town, the prices for parking are \$3 for up to one hour, \$4 for up to two hours, \$5 for up to three hours and \$6 for more than three hours. The automatic parking ticket machine takes only \$1 and \$2 coins, no change is given, and once a parking ticket has been issued the machine returns to the state of \$0 credit. As soon as the amount deposited is \$6 or more, the machine automatically issues a parking ticket.

(a) What are the states of this finite-state automaton?

(b) What are the input symbols of this finite-state automaton?

(c) What is the initial state of this finite-state automaton?

(d) What are the accepting states of this finite-state automaton?

(e) Draw a transition diagram to represent this finite-state automaton.

(f) Describe the language accepted by this finite-state automaton.

3. Let $\times_n$ denote the binary operation of multiplication modulo n.

(a) Write out the Cayley table for the group $(\{1, 2, 3, 4\}, \times_5)$.

(b) Show that $(\{1, 2, 3, 4\}, \times_5)$ is an abelian group. (You may refer to your Cayley table in your explanation and you may assume that $\times_n$ is associative.)

(c) Show that the set $\{1, 2, 3\}$ with the binary operation $\times_4$ is not a group.

4. Let $+_{12}$ denote the binary operation of addition modulo 12. Then $(\mathbb{Z}_{12}, +_{12})$ is a group (you do not need to show this).

(a) Show that $(\{0, 3, 6, 9\}, +_{12})$ is a subgroup of $(\mathbb{Z}_{12}, +_{12})$.

(b) Find the cyclic subgroup of $(\mathbb{Z}_{12}, +_{12})$ generated by 4.

(c) Show that $(\{0, 5, 10\}, +_{12})$ is not a subgroup of $(\mathbb{Z}_{12}, +_{12})$.

5. (Bonus Question, Section 7.6)
Let $3\mathbb{Z} = \{n \in \mathbb{Z} \mid n = 3k, \text{ for some integer } k\}$; that is, $3\mathbb{Z}$ is the set of integers that are divisible by three. Prove that $3\mathbb{Z}$ is countable.
(You may assume that $\mathbb{Z}$ is countable.)