

2. (a) (4 marks) Determine whether the following argument is valid. Please show your working, and let
 s denote "today is Sunday"
 e denote "I do have an exam"
 ℓ denote "I am lucky".

If today is Sunday, then I do not have an exam. $s \rightarrow \neg e$
 If I do not have an exam, then I am lucky. $\neg e \rightarrow \ell$
 I am lucky and today is not Sunday. $\ell \wedge \neg s$
 Therefore I do have an exam. e

Argument: $[(s \rightarrow \neg e) \wedge (\neg e \rightarrow \ell) \wedge (\ell \wedge \neg s)] \rightarrow e$

Assume the argument is invalid (so $T \rightarrow F$).

Then e is False, ℓ is True and s is False

So $s \rightarrow \neg e$ is True, $\neg e \rightarrow \ell$ is True, $\ell \wedge \neg s$ is True and e is False.

The argument is

Invalid

(insert either valid or invalid).

- (b) (3 marks) Give the negation of

$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})$ such that $xy > 0$.

$(\exists x \in \mathbb{R})$ such that $(\forall y \in \mathbb{R}), xy \leq 0$.

Is the original statement true or false? (Write true or false in the box.)

The original statement is

False

(consider $x = 0$.)

3. (9 marks) Let $T = \{\emptyset\}$, $Q = \{T\}$, $R = \{T, \emptyset\}$, $S = \{T, Q, R\}$.
 (Here \emptyset denotes the empty set and $P(X)$ denotes the power set of X .)

- (a) True or false? (Write true or false in the boxes.)

(i) $Q \subseteq R$	True
(ii) $R \subseteq S$	False
(iii) $\emptyset \in T$	True
(iv) $\emptyset \subseteq T$	True

- (b) Write the following sets in the boxes provided, and remember your braces { ... }.

(i) $T \cup Q =$	$\{\emptyset, T\}$
(ii) $T \cap R =$	$\{\emptyset\}$
(iii) $T - R =$	$\{\}$ (or you could say $T - R = \emptyset$)
(iv) $P(T) =$	$\{\emptyset, \{\emptyset\}\}$
(v) $ P(Q) =$	2
(vi) $S \cup R =$	$\{\emptyset, T, Q, R\}$
(vii) $T \times Q =$	$\{(\emptyset, T)\}$