(b) Find an integer x and an integer y which satisfy the following linear diophantine equation.

$$533x + 117y = 65.$$

First find gcd(533, 117).

$533 = 4 \cdot 117 + 65$

$117 = 1 \cdot 65 + 52$

$65 = 1 \cdot 52 + 13$

$52 = 4 \cdot 13 + 0$

$\therefore \gcd(533, 117) = 13.$

Since $13 \mid 65$, a solution exists.

$$\begin{aligned}13 &= 65 - 52 \\ &= 65 - (117 - 65) \\ &= 65(2) - 117 \\ &= (533 - 4 \cdot 117) \times 2 - 117 \\ &= 533(2) + 117(-9).\end{aligned}$$

So $533(10) + 117(-45) = 65$

$\therefore x = 10, y = -45$ is one solution.

Question 6 see next page.

6. (6 marks)

(a) Does there exist a simple graph with vertices having the following degrees?
4, 4, 4, 3, 3, 3, 1, 1.

Explain your answer carefully.

If such a graph existed, it would have total degree $4+4+4+3+3+3+1+1 = 23$. But the total degree of a graph must be even, so no such graph exists.

(b) (i) A tree with 10 vertices has $\boxed{9}$ edges. *(Fill in the box.)*

(ii) A tree has 10 vertices with degrees 1, 1, 1, 1, 1, 1, 2, 2, a, b.
If a, b are 4 or greater, find a and b.

Total degree of a ~~graph~~ tree with 10 vertices (9 edges) is $9 \times 2 = 18$.

$\therefore 1+1+1+1+1+1+2+2+a+b = 18$

so $a + b = 8$

Since each of a, b is 4 or greater, we must have $a = 4$, $b = 4$.

Question 7 see next page.