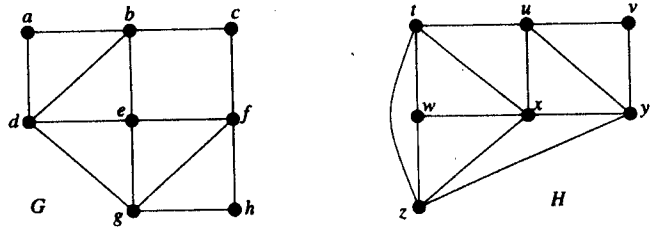
7. (5 marks) For each of the following two graphs G and H, either find an Euler circuit in that graph, or else explain why an Euler circuit does not exist in that graph.



G: An Euler circuit is:

a, b, d, e, b, c, f, e, g, f, h, g, d, a

H: H does not have an Euler circuit since it has two vertices of odd degree.

8. (7 marks)

(a) Evaluate $\dfrac{4 \times \binom{7}{4}}{\binom{6}{3}}$. *Please show your working.*

$$4 \times \frac{7!}{4!3!} \times \frac{3!3!}{6!} = 4 \times \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \times \frac{3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4} = 7$$

(b) A coin is tossed 6 times. In each case the outcome (H for heads or T for tails) is recorded. (One possible outcome is for example, HTTHHH.)

(i) What is the total number of possible outcomes of this coin-tossing experiment?

There are 2 possibilities for each toss, so $2^6 = 64$ possible outcomes.

(ii) In how many of the possible outcomes are exactly 4 tails obtained?

Choose the four positions for the tails.

$\binom{6}{4} = 15$ possible outcomes in which exactly 4 tails are obtained.

(iii) In how many of the possible outcomes are at least 3 heads obtained?

The possibilities are 3 heads, 4 heads, 5 heads or 6 heads.

$\therefore \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 20 + 15 + 6 + 1 = 42.$

(iv) What is the probability that at least 3 heads are obtained?

$$P(\text{at least 3 heads}) = \frac{42}{64} = \frac{21}{32} = 0.65625$$