9. (6 marks) A bag contains 11 distinct letters: A, B, C, D, E, F, G, H, I, J, K.

(a) How many different ways are there to choose 3 letters all at once from the bag of 11 letters?

$\binom{11}{3} = 165$

There are 165 ways to choose 3 letters from the bag of 11.

(b) Now the 3 letters are picked one at a time, and the order of choice is noted, as a 3-letter "word". How many different 3-letter "words" can be chosen from the bag of 11 letters?

$11 \times 10 \times 9 = 990$

There are 990 different 3-letter "words".

(c) There are three vowels in the bag (A, E and I). When 3 letters are chosen from the bag of 11 letters all at once, what is the probability that exactly one of the 3 letters is a vowel?

Choose one vowel and two consonants:

$\binom{3}{1} \times \binom{8}{2} = 3 \times 28 = 84$

$P(\text{exactly one is a vowel}) = \frac{84}{165} \approx 0.51.$

10. (8 marks)

Let $X = \{x \in \mathbf{Z} \mid -1 \le x \le 4\}$ and $Y = \{y \in \mathbf{Z} \mid 0 \le y \le 5\}$.

Define $f : X \to \mathbf{Z}$ by $f(x) = |x| + 1$ and $g : Y \to \mathbf{Z}$ by $g(y) = \left\lfloor \frac{y}{2} \right\rfloor + 1$.

(i) The range of f is $\{1, 2, 3, 4, 5\}$

(ii) The range of g is $\{1, 2, 3\}$

(iii) Is f one-to-one? Explain your answer.

f is not one-to-one.

$f(-1) = f(1)$ but $-1 \neq 1$.

(iv) Is Y a subset of the range of f? Explain your answer.

No. $0 \in Y$ but $0 \notin$ range of f,

so Y is not a subset of the range of f.

(v) Calculate $(g \circ f)(2)$ and also $(f \circ g)(2)$.

$(g \circ f)(2) = g(f(2)) = g(3) = 2$

$(f \circ g)(2) = f(g(2)) = f(2) = 3$

(vi) Write general expressions, in terms of x, for the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(f \circ g)(x) = f(g(x)) = f\left(\left\lfloor \frac{x}{2} \right\rfloor + 1\right) = \left|\left\lfloor \frac{x}{2} \right\rfloor + 1\right| + 1.$

$(g \circ f)(x) = g(f(x)) = g(|x| + 1) = \left\lfloor \frac{|x|+1}{2} \right\rfloor + 1$