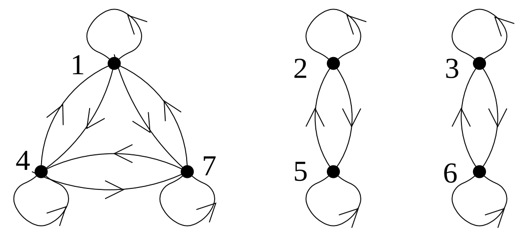
Relations

Equivalence Relations

An equivalence relation R on a set A partitions the set A into subsets in such a way that the elements within a subset are equivalent. The directed graph of an equivalence relation has separate components and within each component all the possible directed edges occur. Formally, a relation R is an equivalence relation if, and only if, R is reflexive, symmetric and transitive.

Example: Let A be the set $\{1, 2, 3, 4, 5, 6, 7\}$ and let R be the relation

$$m\ R\ n \text{ if and only if } 3 \mid (m - n).$$

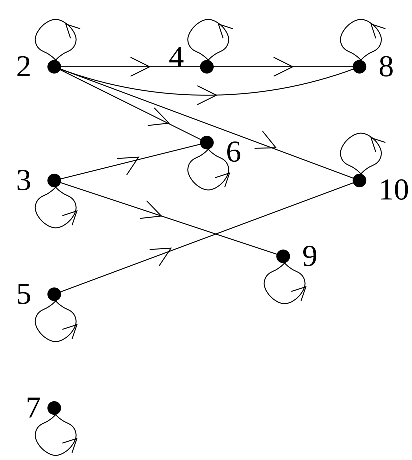


Partial Orders

A partial order is a relation S on a set A that places some of the elements of A in order. The ordering has the property that if a comes before b and b comes before c, then a comes before c, but there may be some elements d and e for which neither d comes before e nor e comes before d. The directed graph of a partial order S can be drawn in such a way that all the arrows joining distinct vertices go in one direction. Formally, a relation S is a partial order if, and only if, S is reflexive, anti-symmetric and transitive.

Example: Let A be the set $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let S be the relation

$$m\ S\ n \text{ if and only if } m \text{ is a factor of } n.$$



Total Orders

A total order is a relation T on a set A that places all the elements of A in order. The ordering has the property that if a comes before b and b comes before c, then a comes before c, and every element of A occurs in a single chain. The directed graph of a total order T can be drawn in such a way that all the elements of A are in a line and all the arrows joining distinct vertices go in the same direction along the line. Formally, a relation T is a total order if, and only if, T is reflexive, anti-symmetric, transitive, and $\forall a, b \in A$, either $a \; T \; b$ or $b \; T \; a$.

Example: Let A be the set $\{1, 2, 3, 4, 5\}$ and let T be the relation

$$m \; T \; n \text{ if and only if } m \leq n.$$