INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Sample Semester Examination,

MATH1061 (Ipswich) Sample Exam

DISCRETE MATHEMATICS

(Unit Courses, B. Inf. Tech.)

Time: **TWO Hours** for working

Ten minutes for perusal before examination begins

Check that this examination paper has 15 printed pages!

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION PAPER!

Candidates may bring one A4 sheet of handwritten notes into the examination.
Candidates should attempt all questions.
The allocated marks for each question are shown. There are 100 marks allocated on this examination.
Use the blank pages for rough work, amendments etc.
Pocket calculators allowed.

FAMILY NAME (PRINT): _____

GIVEN NAMES (PRINT): _____

STUDENT NUMBER: | | | | | | | | |

SIGNATURE: _____

EXAMINER'S USE ONLY					
QUESTION	MARK	QUESTION	MARK	QUESTION	MARK
1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10		15	
				TOTAL	

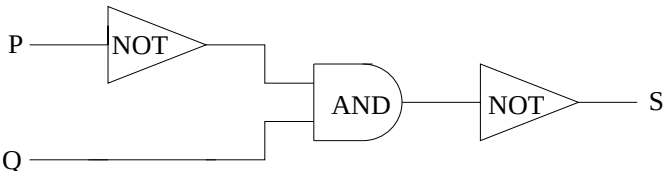
1. **(a)** Use a truth table to determine whether the following statement form is a tautology, a contradiction or neither.

$$((p \wedge q) \rightarrow (p \vee q)) \;\leftrightarrow\; (\sim p \vee \sim q).$$ (3 marks)

p	q	

This statement form is _____

(b) Consider the following digital logic circuit.



(i) Complete the input/output table for this circuit. (2 marks)

P	Q	S
1	1	
1	0	
0	1	
0	0	

(ii) Use your input/output table to draw a simpler circuit which is equivalent to the circuit given above.

(2 marks)

Question 2 see next page.

2. Consider the statement

$$\forall r \in \mathbb{R}, \text{if } r \text{ is irrational, then } r^2 \text{ is irrational.}$$

(a) Write the contrapositive of this statement. (2 marks)

(b) Write the negation of this statement. (2 marks)

(c) State which is true, the original statement or the negation. (1 mark)

3. Prove that the following statement is true.

For all integers a and b, if $4 \mid a$ and $4 \mid b$, then $4 \mid (a+b)$.

(4 marks)

Question 4 see next page.

4. Use mathematical induction to prove that the following statement is true.

For all integers $n \geq 3$, $\quad n^2 > 2n + 1$.

(6 marks)

Question 5 see next page.

5. **(a)** Use the Euclidean Algorithm to find $\gcd(423, 81)$. (2 marks)

(b) Determine whether or not there exist integers x and y that satisfy the following linear diophantine equation. If so, then find one such pair of integers. If not, explain why not.

$$81x + 423y = 36$$

(2 marks)

Question 6 see next page.

6. Let $U = \{10, 11, 12, 13, 14, 15\}$ be a universal set with subsets

$$A = \{x \in U \mid x \text{ is even}\} \text{ and } B = \{x \in U \mid x \text{ is divisible by } 3\}.$$

Determine the elements in each of the following sets and write each set in the space provided.

(5 marks)

$A \cap B$ =

$A \cup B$ =

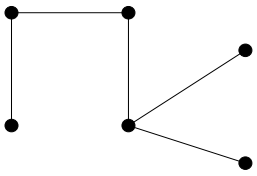
$A - B$ =

A^c =

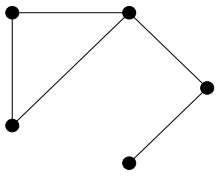
$\mathcal{P}(B)$ =

7. Determine whether or not each of the following graphs is a tree. Give a brief justification for your answers.

(2 marks)



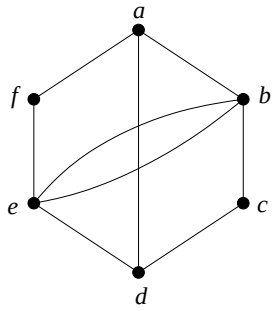
(1)



(2)

Question 8 see next page.

8. Let G be the following graph.



(a) Determine whether or not G has an Euler circuit. If it does, give one such circuit; if it does not, explain why not.

(2 marks)

(b) Determine the total degree of G. (2 marks)

(c) Determine whether or not G has a subgraph that has four vertices and five edges. Justify your answer by drawing the subgraph or by explaining why such a subgraph does not exist.

(2 marks)

Question 9 see next page.

9. Let ρ and σ be relations on the set $A = \{1, 2, 3\}$ defined as follows:

$$\rho = \{(1,3),\ (1,1),\ (2,1),\ (2,2),\ (3,3)\}$$

and

$$\sigma = \{(1,2),\ (1,3),\ (2,1),\ (1,1),\ (2,2),\ (2,3)\}.$$

(a) Draw the directed graphs that correspond to ρ and σ. (2 marks)

(b) Write the elements of the relation σ^{-1} in the box below.

(2 marks)

σ^{-1}	

Question 9(c) see next page.

9. (c) Let ρ and σ be the relations on the set $A = \{1, 2, 3\}$ as defined on the previous page; that is,

$$\rho = \{(1,3),\ (1,1),\ (2,1),\ (2,2),\ (3,3)\}$$

and

$$\sigma = \{(1,2),\ (1,3),\ (2,1),\ (1,1),\ (2,2),\ (2,3)\}.$$

For each of the relations ρ and σ, state whether or not it is (i) reflexive, (ii) symmetric, (iii) antisymmetric, (iv) transitive? Write your answer in the appropriate box below, with a brief justification.

(8 marks)

	ρ	σ
(i) reflexive?		
(ii) symmetric?		
(iii) antisymmetric?		
(iv) transitive?		

Question 10 see next page.

10. Let S be a relation on the set $\mathbb{Z}$ defined as follows

$$a \, S \, b \quad \text{if and only if} \quad a \leq b.$$

(a) Prove that S is an partial order relation. (5 marks)

(b) Explain why S is a total order relation. (2 marks)

Question 11 see next page.

11. Let f and g be functions defined as

$$f : \mathbb{R} \to \mathbb{R} \text{ such that } f(x) = 3x + 6,$$
$$g : \mathbb{R} \to \mathbb{R} \text{ such that } g(x) = x^2 - 2.$$

(a) Is the function f one-to-one and/or onto? Justify your answer. (5 marks)

(b) Is the function g one-to-one and/or onto? Justify your answer. (5 marks)

(c) Calculate $(g \circ f)(x)$ and $(f \circ g)(x)$. (4 marks)

Question 12 see next page.

12. **(a)** Consider the systems $(\mathbb{Z}_7 - \{0\}, \odot)$, $(\mathbb{Z}_6 - \{0\}, \otimes)$, $(\mathbb{Z}_n, \oplus)$, $(\mathbb{R}, \times)$. Complete the following table with ticks (for yes) and crosses (for no) to indicate whether the given system is a group. Note that $\odot$ denotes multiplication modulo 7, $\otimes$ denotes multiplication modulo 6, $\oplus$ denotes addition modulo n and $\times$ denotes multiplication.

(4 marks)

	$(\mathbb{Z} - \{0\}, \odot)$	$(\mathbb{Z}_6 - \{0\}, \otimes)$	$(\mathbb{Z}_n, \oplus)$	$(\mathbb{R}, \times)$
Group?				

(b) For each system in part (a) which is not a group give a brief explanation of why it is not a group.

(3 marks)

Question 13 see next page.

13. **(a)** Construct the Cayley table for the group $(\mathbb{Z}_5 - \{0\}, \odot)$ where $\odot$ is the binary operation of multiplication modulo 5.

(3 marks)

(b) Is $(\mathbb{Z}_5, \oplus, \odot)$ a field? Recall that $\oplus$ is the binary operation of addition modulo 5 and $\odot$ is the binary operation of multiplication modulo 5. Justify your answer.

(3 marks)

Question 14 see next page.

14. **(a)** How many distinct arrangements can be made from the letters of the word MISSISSIPPI?

(2 marks)

(b) How many distinct arrangements can be made from the letters of the word MISSISSIPPI that begin with an S and end with an M?

(2 marks)

(c) How many distinct arrangements can be made from the letters of the word MISSISSIPPI which contain the two letters PM next to each other in the given order?

(2 marks)

Question 15 see next page.

15. **(a)** Evaluate $\dfrac{5 \times \binom{9}{4}}{\binom{8}{4}}$. Show your working. (2 marks)

(b) A regular 6-sided die is rolled five times. In each case the result (1, 2, 3, 4, 5 or 6) is recorded. The order of the five results is also recorded. For example, one possible outcome is 3 4 6 1 3.

(i) What is the total number of possible outcomes of this die-rolling experiment?

(1 mark)

(ii) In how many of the possible outcomes is the first result a three and exactly two threes are obtained in total?

(2 marks)

(iii) In how many of the possible outcomes is the first result a three and at least two threes are obtained in total?

(2 marks)

(iv) What is the probability of obtaining an outcome in which there are no fives or sixes?

(2 marks)