

Corrected solution to Sample Exam Question 15(b)

15. (b) A regular 6-sided die is rolled five times. In each case the result (1, 2, 3, 4, 5 or 6) is recorded. The order of the five results is also recorded. For example, one possible outcome is 3 4 6 1 3.

- (i) What is the total number of possible outcomes of this die-rolling experiment?

(1 mark)

There are 6 possible results for each roll, so there are $6^5 = 7776$ possible outcomes.

- (ii) In how many of the possible outcomes is the first result a three and exactly two threes are obtained in total?

(2 marks)

The possible outcomes (where * is one of 1, 2, 4, 5 or 6) are:

3 3 * * *, 3 * 3 * *, 3 * * 3 *, and 3 * * * 3.

There are 5^3 possibilities for each of these four cases, so the total number of possible outcomes of this type is $4 \cdot 5^3 = 500$.

- (iii) In how many of the possible outcomes is the first result a three and at least two threes are obtained in total?

(2 marks)

This is equal to the number of outcomes in which the first result is a 3 minus the number of outcomes in which the first 3 is the only 3.

This is $6^4 - 5^4 = 671$.

This could also be calculated as those outcomes in which the first result is a three and exactly 2 3s are obtained plus exactly 3 3s are obtained plus exactly 4 3s are obtained plus exactly 5 3s are obtained. This is $4 \cdot 5^3 + 6 \cdot 5^2 + 4 \cdot 5 + 1 = 671$.

- (iv) What is the probability of obtaining an outcome in which there are no fives or sixes?

(2 marks)

The number of outcomes with no fives or sixes is $4^5 = 1024$.

The probability of obtaining an outcome that has no fives or sixes is

$$\frac{1024}{7776} = \frac{32}{243}.$$