

3. Write the following argument in symbolic form and then show that the argument is invalid.

If you studied and you are not tired, then you will pass this test.

You are tired but you will pass this test. Hence you studied.

(6 marks)

Let s represent "you studied".

Let t represent "you are tired".

Let p represent "you will pass this test."

The argument can be written symbolically as:

$$([(s \wedge \sim t) \rightarrow p] \wedge (t \wedge p)) \rightarrow s.$$

For this argument to be invalid, we must find truth values that make each premise true and the conclusion false.

If s is False, t is True and p is True,
then $(s \wedge \sim t) \rightarrow p$ is true (since $F \rightarrow T$ is T)
 $(t \wedge p)$ is true

and s is false,

so the argument is invalid.

| s | t | p | $(s \wedge \sim t)$ | $(s \wedge \sim t) \rightarrow p$ | $(t \wedge p)$ |
|-----|-----|-----|---------------------|-----------------------------------|----------------|
| T | T | T | F | T | T |
| T | T | F | F | T | F |
| T | F | T | T | T | F |
| T | F | F | T | F | F |
| F | T | T | F | T | T |
| F | T | F | F | T | F |
| F | F | T | F | T | F |
| F | F | F | F | T | F |

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The critical rows are marked with a *.

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In this critical row, the premises are true and the conclusion is false.

∴ The argument is invalid.