4. Consider the statement

$$\forall x \in \mathbb{R}, \text{if } x^2 \geq 0, \text{ then } x \geq 0.$$

(a) Write the contrapositive of this statement, in symbolic form. (2 marks)

$\forall x \in \mathbb{R}$, if $x < 0$ then $x^2 < 0$.

(b) Write the negation of this statement, in symbolic form. (2 marks)

$\exists x \in \mathbb{R}$ such that $x^2 \geqslant 0$ and $x < 0$.

5. Prove the following statement.

For all integers a, b and c, if $a \mid b$ and $a \mid c$, then $a^2 \mid bc$.

(5 marks)

Proof

Suppose that a, b, c are integers and that $a \mid b$ and $a \mid c$.

Since $a \mid b$, $b = ak$ for some integer k.

Since $a \mid c$, $c = a\ell$ for some integer ℓ.

Thus $b \cdot c = (ak)(a\ell) = a^2(k\ell)$.

Now $k\ell$ is an integer so
$bc = a^2 m$ for some integer m.

$\therefore a^2 \mid bc$.