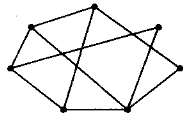
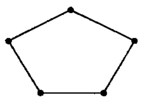
6. Let G and H be the two graphs shown below.

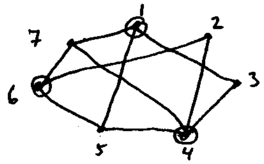


G



H

(a) Show that the graph G is bipartite. (2 marks)



The vertices of G can be partitioned into two sets $\{1,4,6\}$, $\{2,3,5,7\}$ so that each edge joins a vertex from one set to a vertex in the other set.

$\therefore$ G is bipartite

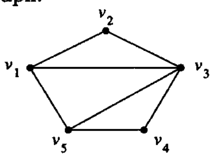
(b) Use part (a) to explain why the graph H is not a subgraph of the graph G. (2 marks)

H is <u>not</u> bipartite

← problem!

$\therefore$ H cannot be a subgraph of G.

7. Let G be the following graph.



Determine whether the following statements are true or false and circle the appropriate answer. (3 marks)

(a) The walk $v_1, v_2, v_3, v_4, v_5, v_3, v_1$ is a path. True False

(b) The walk $v_1, v_2, v_3, v_4, v_5, v_3, v_1$ is a simple path. True False

(c) The walk $v_1, v_2, v_3, v_4, v_5, v_3, v_1$ is a circuit. True False

(d) The walk $v_1, v_2, v_3, v_4, v_5, v_3, v_1$ is a simple circuit. True False

(e) The walk $v_1, v_2, v_3, v_4, v_5, v_3, v_1$ is an Euler circuit of G. True False

(f) The graph G has an Euler circuit. True False

badly worded question so both answers were marked correct

8. **(a)** Determine the total degree of a tree with 10 vertices. (2 marks)

A tree with 10 vertices has 9 edges.

$\therefore$ The total degree is $2 \times 9 = 18$.

(b) Let T be a tree with 10 vertices in which there is exactly one vertex of degree four and exactly five vertices of degree one. Determine the degrees of the other vertices of T and draw one such tree. (4 marks)

Let the degrees of the 10 vertices be $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}$.

$18 = 4+1+1+1+1+1+d_7+d_8+d_9+d_{10}$

and each of $d_7, d_8, d_9, d_{10} \geq 2$.

$9 = d_7+d_8+d_9+d_{10}$

$\therefore$ one vertex must be degree 3 and the other three vertices must be degree 2.

One such tree is: