9. Let $A = \{1,2,3,4\}$ and let R be the binary relation on A given by the following ordered pairs.

$$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$$

Give a brief reason for each of your answers to the following questions.

(7 marks)

(a) Is R reflexive?

Yes. $(1,1), (2,2), (3,3), (4,4)$ are all in R.

(b) Is R symmetric?

No. $(1,2) \in R$ but $(2,1) \notin R$.

(c) Is R anti-symmetric?

Yes. For all distinct $a, b \in A$,
if $(a,b) \in R$ then $(b,a) \notin R$.

(d) Is R transitive?

Yes. For all $a,b,c \in A$,
if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$.

(e) Is R an equivalence relation?

No. R is not symmetric.

(f) Is R a partial order?

Yes. R is reflexive, anti-symmetric and transitive.

(g) Is R a total order?

Yes. For all $a,b \in A$,
either $(a,b) \in R$ or $(b,a) \in R$.

10. Let S be the binary relation on the set of integers defined by

$m \; S \; n$ if, and only if, $m - n$ is odd.

(5 marks)

(a) Is S reflexive? Either prove that it is, or give a counterexample to show that it is not.

No. 2 is an integer but 2 is not related to itself since $2-2=0$ which is <u>not</u> odd.

(b) Is S symmetric? Either prove that it is, or give a counterexample to show that it is not.

Yes. $\forall m,n \in \mathbb{Z}$, if mSn then nSm.
Suppose that m,n are integers and mSn.
Then $m-n$ is odd, so $m-n = 2k+1$ for some integer k
Hence $n-m = -(2k+1) = 2(-k)-1$, which is odd.
(The negative of an odd integer is odd.)
$\therefore nSm$.

(c) Is S transitive? Either prove that it is, or give a counterexample to show that it is not.

No. 2, 3 and 6 are integers.
6 is related to 3 since $6-3=3$. (odd)
3 is related to 2 since $3-2=1$. (odd)
but 6 is not related to 2. since
$6-2=4$ which is <u>not</u> odd.