# Interactive Discrete Mathematics

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## 1. Introduction

It is my experience that a considerable amount of time and expertise is being focused on new technologies and their use in the teaching of calculus. By contrast discrete mathematics has not been given the same attention. In the Department of Mathematics, at The University of Queensland, we teach discrete mathematics to large groups of first year Electrical Engineering and Information Technology students. In the past we have used traditional methods, 2 lectures & 1 tutorial to deliver the subject material. In 1999, we resolved to change our teaching mode to allow students greater flexibility in their learning activities. Our charter was to develop a directed learning facility that placed the knowledge–acquisition process in the domain of the students. The aim of this paper is to report on the progress in this area and thus to promote a discussion of the development of web based material for discrete mathematics. I will document the problems and solutions we encountered in the hope that this will generate interest in the area, thus fuelling the development of valuable interactive learning materials for use in the teaching of discrete mathematics.

## 2. The fundamentals

It is our belief that the best resource available to students is a good textbook. Thus our strategic plan involved selecting a good textbook [**2**] and then providing valuable resources to complement the book. We were aware that the present day students have very little experience in reading mathematics and have great difficulty navigating an 800 page textbook. Thus we developed a workbook to complement our text. We divided the material into bit size pieces and provided the students with a timetable, based on weekly cycles, for reviewing the material. At the beginning of each section we endeavoured to provide the students with a concise rationale for the material to be reviewed, but at the same time we wanted to capture their attention as is indicated in the following excerpt from the workbook.

**The Logic of Compound Statements**

In the first chapter covered in this course, you will be introduced to formal Logic. The ability to think logically and to determine whether or not an argument is valid is a vital skill. Consider the following excerpt from *Alice in Wonderland*, by Lewis Carroll:

> "Do you mean that you think you can find out the answer to it?" said the March Hare.
> "Exactly so," said Alice.
> "Then you should say what you mean," the March Hare went on.
> "I do," Alice hastily replied; "at least–at least I mean what I say–that's the same thing, you know."
> "Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

In order to win arguments and debates it is extremely important that you can determine whether the given arguments are valid. In this chapter we shall learn how to rewrite a propositional argument in an abstract form and use truth tables to determine the validity of the argument.
Logic is also very important in areas of computer science. The design of digital logic circuits depends directly on the abstract logical connectives which you will find in this chapter. The last two sections of this chapter discuss the applications of logic to computer science.

The workbook provides *Learning objectives* at the beginning of each section and concludes each section with *Special Points* and a *Checklist.* Throughout the workbook, we carefully stated the prescribed reading for the week, but more importantly we endeavoured to identify the important definitions and associated theorems. To ensure that the students noted this material, we required them to transcribe the material into specified sections of the workbook. The following exerpt indicates how this was done.

**Exercise 1.1.1: Definitions**
Fill in the blanks to define the following terms.

1. A **statement** or **proposition** is ________

   ____________________

2. If $p$ is a statement variable, the **negation** of $p$ is ____________________

   It has the ________ truth value from $p$: if $p$ is true, then $\sim p$ is ________

   if $p$ is false, then $\sim p$ is ________

The chronology of definitions provided the students with good reference material and a good revision summary. To develop the students mathematical ability we included strategically placed exercises to reinforce the theory.

## 3. The Web

The workbook was made available to the student in hard copy but was also placed on the web [**1**]. Full details of all definitions and theorems and solutions to all exercises were also placed on the web. However, we realised that to simply store the text in HTML format was a futile exercise as a printed copy would serve the students better. We recognised that the web can provide easy and speedy access to information as well as an interactive environment to engage the students. So with this in mind we created a dynamic navigation system that allowed easy movement between chapters and sections and encompassed the following features.

*A Reference Section:* A popup help screen, linked to the menu system, has been developed to provide access to all important definitions and theory. The definitions are sorted according to topics and a *Contents* guide is given. The *Contents* provide a preliminary search mechanism, while a *Search* facility on key words allows for a finer and more accurate searching tool. In this way the students have fast and accurate access to all the theory. In addition, the definitions etc are given in precise terminology and not buried in the general discussion of the subject. This material has been presented in a printable format for those students who prefer this.

*Interactive Problem Solver:* Students can be access solutions to all problems posed in the workbook. Solutions are available in their totality, or can accessed through hints to guide the development of a student's own solution. In addition, if students wishes they may attempt to solve the problem on line and their solutions are marked in real time. This facility is demonstrated in the excerpt given below. This excerpt shows a part of a typical question in which students are asked to rewrite a statement symbolically using logical connectives. Students enter their solution by simply clicking on the appropriate letter or symbol displayed above the solution field and the symbol automatically appears in the solution box.

**Exercise 1:** Rewrite the following statements (in symbolic form), then check your result.
**a)** Either Sam will come to the party and Max will not, or Sam won't come to the party and Max will enjoy himself at the party.
Let $p$ be "Sam will come to the party", let $q$ be "Max will come to the party", and let $r$ be "Max will enjoy himself at the party".

[p] [q] [r]
[$\sim$] [$\wedge$] [$\vee$] [(] [)] [Delete]
[ ] [Clear All]
[Hint] [Solution] [Check]

Students can amend their solution using the *delete* function, *check* their answers at any stage, *clear all* and start over or at any stage access *hints* or the *full solution.* One of the problems we encountered was the displaying of mathematical symbols in a HTML environment. This was a major difficulty as checking facilities usually require precise syntax. In a mathematical environment, this feature normally makes the entering and checking of data an onerous task. However, the ingenious inclusion of a symbolic toolpad above each question means that the solution can be displayed by clicking on the appropriate symbols in the correct order. This is an extremely user friendly environment, the mechanics of which are easily grasped.

Another example of the interactive tools we have developed is the dynamic *Truth Table Solver.* To enter information the student need only click on boxes, in a similar fashion to that used in the mouse driven "Microsoft Minesweeper" game. To enter data the students click on the appropriate box until the required $T$ or $F$ appears. In this way the students can easily construct a truth table without worrying about upper and lower case or whether the information should be entered as $T$ and $F$ or 1 and 0. A typical exercise is given below.

**Exercise 5:** Are the statement forms $p \wedge \sim q$ and $(p \vee q) \wedge \sim q$ logically equivalent? There are two statement variables so the truth table will have 4 rows.

| $p$ | $q$ | $\sim q$ | $p \vee q$ | $p \wedge \sim q$ | $(p \vee q) \wedge \sim q$ |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Check   Clear All   Help

Separation of the truth table into individual fields implies that when the students solution is marked the exact error is identified. However to ensure that we have anticipated all difficulties an online *Help system* documenting the facilities of the *On Line Problem Solver* has been included

## 4. Future Development

It is envisaged that in the future we will include animations which illustrate some of the key aspects of the theory. It is hoped that this facility will aid the students understanding in much the same way a lecturer explains difficult concepts through the use of illumniating diagrams.

## 5. Development Tools

A variety of programming tools have been used in the development of this material, these include Visual InterDev, Fireworks, Flash and Visual Cafe. In addition, where necessary certain aplets, etc, have been developed from pure JavaScript or HTML script. In all cases the material is completely platform–independent. This is a significant factor given the wide user base.

## 6. Access

A prototype of the webpage can be viewed at `http://www.maths.uq.edu.au/~dmd`.

## References


1. Donovan, Diane, MT161 Course Material, (available from `http://www.maths.uq.edu.au`).
2. Epp, Susanna S., Discrete Mathematics with Applications, Brooks/Cole Publishing Company, Boston, 1995.