Exercise Set 1.1

Q26

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

No the given statements forms are not logically equivalent.

Q35

The statement can be rewritten as $-2 < x$ and $x < 7$. So let p represent $-2 < x$ and q represent $x < 7$. Then $-2 < x < 7$ is the statement form $p \wedge q$. By DeMorgan's Law $\sim (p \wedge q) \equiv \sim p \vee \sim q$ and $\sim p$ represents $-2 \geq x$ and $\sim q$ represents $x \geq 7$. So the negation of $-2 < x < 7$ is $-2 \geq x$ or $x \geq 7$, or equivalently $x \leq -2$ or $x \geq 7$.

Q43

p	q	r	$((\sim p \wedge q)$	$\wedge$	$(q \wedge r))$	$\wedge$	$\sim q$
T	T	T	F	F	T	F	F
T	T	F	F	F	F	F	F
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	F	F
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

This is a contradiction as the statement form takes the value F (false) in all cases.

Exercise Set 1.2

Q10

p	q	r	$(p \to r)$	$\leftrightarrow$	$(q \to r)$
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Q20e e) Let n represent the statement x is non-negative, p represent the statement x is positive, and o represent the statement x is 0. Then 'If x is non-negative, then x is positive or x is 0' is equivalent to $n \to (p \vee o)$.

So
$$\begin{aligned} n \to (p \vee o) &\equiv \sim n \vee (p \vee o) \\ \sim (n \to (p \vee o)) &\equiv \sim (\sim n \vee (p \vee o)) \equiv n \wedge \sim (p \vee o) \\ &\equiv n \wedge (\sim p \wedge \sim o) \end{aligned}$$

Q21bc If $p \to q$ is false then p is true and q is false.

b) So $p \vee q$ has truth value 'true'.

c) And $q \to p$ has truth value 'true'.

Exercise Set 1.3

Q32 Let

c represent the statement *I get a Christmas bonus.*

s represent the statement *I will buy a stereo.*

m represent the statement *I will sell my motorcycle.*

Then the argument is equivalent to

$$((c \to s) \wedge (m \to s)) \to ((c \vee m) \to s)$$

Assume that the given statement can take a false value. Hence assume the premise is true and the conclusion is false. So $(c \to s) \wedge (m \to s)$ is true, and $(c \vee m) \to s$ is false.

Since $c \vee m \to s$ is false, $c \vee m$ is true and s is false.

Since $(c \to s) \wedge (m \to s)$ is true, $c \to s$ is true and $m \to s$ is true. But from the previous step s is false, so both c and m must be false. But we now have c and m false, and $c \vee m$ true. This is a contradiction so the original argument must be valid.