

Exercise Set 2.1

Q21d $\forall x, y \in \mathbb{Z}$ if x and y are odd then $x.y$ is odd.

Q22b $\forall x$, if x is a computer science student, then x needs to take data structures. \forall computer science students x , x needs to take data structures.

Exercise Set 2.2

Q15d True. If it were false we would need to exhibit a number belonging to D which is of the form $y2$, where y is replaced by a digit other than 3 or 4. But the only element of D which is of the form $y2$ is 32. So we cannot find a counter example. Thus the statement is true.

Q21 Let p represent the statement n is prime, o represent the statement n is odd, t represent the statement $n = 2$. Then the statement form is $p \rightarrow (o \vee t)$

The negation of this statement form is

$$\begin{aligned} \sim (p \rightarrow (o \vee t)) &\equiv \sim (\sim p \vee (o \vee t)) &&\equiv p \wedge \sim (o \vee t) \\ &&&\equiv p \wedge \sim o \wedge \sim t \end{aligned}$$

Thus the negation is

$\exists n \in \mathbb{Z}$, such that n is prime and n is not odd and $n \neq 2$.

Exercise Set 2.3

Q15 There exists a book, which all people have read.

Negation: \forall books b , \exists a person p such that p has not read b .

Equivalently: Given any book, there exists a person who has not read that book.

Q19 There exists a real number x such that for all real numbers y , $x + y = 0$.

Negation: $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ such that $x + y \neq 0$

Equivalently: Given any real number x there exists a real number y such that $x + y \neq 0$

Q54 b) Let $P(x)$ be the statement that x is positive and $Q(x)$ be the statement that x is negative.

Then $\exists x \in \mathbb{R}, (P(x) \wedge Q(x))$ means there exists a real number which is both positive and negative. This is a false statement (note 0 is neither positive or negative). But $(\exists x \in \mathbb{R}, P(x)) \wedge (\exists x \in \mathbb{R}, Q(x))$ means there exists a real number which is positive and there exists a real number which is negative. This is a true statement, for example, 2 is positive and -100 is negative. Hence these two statements do not have the same truth values.