MATH1061/7861 Semester 2, 2005 Solution Ass. 4

Exercise Set 3.5

Q28 Let $n = 2m + 1$ where $m \in \mathbb{Z}$. Then

$$\frac{n^2}{4} = \frac{(2m+1)^2}{4} = \frac{4m^2+4m+1}{4} = m^2 + m + \frac{1}{4}.$$

Hence $\lfloor \frac{n^2}{4} \rfloor = m^2 + m = m(m+1)$. And

$$\frac{(n-1)}{2}\frac{(n+1)}{2} = \left(\frac{2m+1-1}{2}\right)\left(\frac{2m+1+1}{2}\right) = \left(\frac{2m}{2}\right)\left(\frac{2m+2}{2}\right) = m(m+1).$$

Thus $\lfloor \frac{n^2}{4} \rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$ whenever n is odd.

Exercise Set 3.6

Q First we will prove this result by contradiction. Assume that $a \mid b$ and $a \nmid c$, but $a|(b+c)$. Then we have $b = ak$ and $b + c = am$, where $k, m \in \mathbb{Z}$. Now substituting $b = ak$, into $b + c = am$, gives $ak + c = am$ or $c = am - ak = a(m - k)$. Since $(m - k) \in \mathbb{Z}$, $a \mid c$. But this is a contradiction as we assumed that $a \nmid c$. So if $a \mid b$ and $a \nmid c$ then $a \nmid (b + c)$.

Using the hint given on the question paper we see that to prove the contrapositive we need to prove that

If not $a \nmid (b + c)$ and $a \mid b$ then not $a \nmid c$, or in other words

If $a \mid (b + c)$ and $a \mid b$ then $a \mid c$.

Which is precisely what we showed above.

Exercise Set 3.8

Q1

$3510 = 5 \times 672 + 150$	$150 = 3510 - 5 \times 672$	$-(1)$
$672 = 4 \times 150 + 72$	$72 = 672 - 4 \times 150$	$-(2)$
$150 = 2 \times 72 + 6$	$6 = 150 - 2 \times 72$	$-(3)$
$72 = 12 \times 6.$		

So $gcd(3510, 672) = 6$

Starting with equation (3) we have

$6 = 150 - 2 \times 72,$	and substituting values from eqn (2) gives
$6 = 150 - 2(672 - 4 \times 150).$	So
$6 = 9 \times 150 - 2 \times 672.$	Now substituting values from enq (1) gives
$6 = 9(3510 - 5 \times 672) - 2 \times 672$	or
$6 = 9 \times 3510 - 47 \times 672.$	So $m = 9$ and $n = -47$.

Exercise Set 4.1

Q7 $a_k = 2k + 1$, $b_k = (k-1)^3 + k + 2$

$$\begin{array}{lll} k = 0: & a_0 = 2 \times 0 + 1 = 1 & b_0 = (0-1)^3 + 0 + 2 = -1 + 2 = 1 \\ k = 1: & a_1 = 2 \times 1 + 1 = 3 & b_1 = (1-1)^3 + 1 + 2 = 3 \\ k = 2: & a_2 = 2 \times 2 + 1 = 5 & b_2 = (2-1)^3 + 2 + 2 = 5 \\ k = 3: & a_3 = 2 \times 3 + 1 = 7 & b_3 = (3-1)^3 + 3 + 2 = 13 \end{array}$$

Hence $a_0 = b_0$, $a_1 = b_1$, and $a_2 = b_2$ and $a_3 \neq b_3$.

Q39 $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!} = \sum_{i=1}^{n} \frac{i}{(i+1)!}$

Q43 $\frac{6!}{8!} = \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{8 \times 7} = \frac{1}{56}$

Q44 $\frac{4!}{0!} = \frac{4!}{1} = 4 \times 3 \times 2 \times 1 = 24$

Q50

$$\begin{aligned} \frac{n!}{(n-k+1)!} &= \frac{n(n-1)(n-2)\ldots(n-k+2)(n-k+1)!}{(n-k+1)!} \\ &= n(n-1)(n-2)\ldots(n-k+2) \end{aligned}$$