MATH1061/7861 Semester 2, 2005 Solution Ass. 6

Chapter 5.2

Q. Prove that $A \subseteq B$ if and only if $A \cap B = A$

Let p correspond to $x \in A$ and let q correspond to $x \in B$. Then the above statement is equivalent to $(p \to q) \leftrightarrow ((p \wedge q) \leftrightarrow p)$

The truth table for this statement form is:

p	q	$((p \to q)$	$\leftrightarrow$	$((p \wedge q)$	$\leftrightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	T	F	T
F	F	T	T	F	T

Since the statement form is a tautology the set law is true for all sets A and B.

Exercise Set 5.3

Q20 No, as $S_a = \{\{a\}, \{a,b\}, \{a,c\}\}$, $S_b = \{\{b\}, \{a,b\}, \{b,c\}\}$ and $S_a \cap S_b = \{\{a,b\}\} \neq \emptyset$.

Q1 $A \times B = \{(1,2), (1,3), (2,2), (2,3)\}$

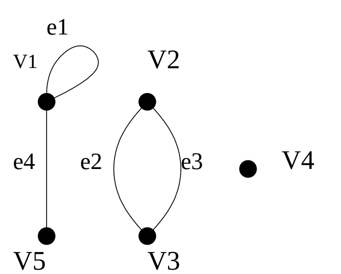
$$\begin{aligned}\mathcal{P}(A \times B) \; = \; & \{\emptyset, \{(1,2)\}, \{(1,3)\}, \{(2,2)\}, \{(2,3)\}, \{(1,2),(1,3)\}, \{(1,2),(2,2)\}, \\ & \{(1,2),(2,3)\}, \{(1,3),(2,2)\}, \{(1,3),(2,3)\}, \{(2,2),(2,3)\}, \\ & \{(1,2),(1,3),(2,2)\}\{(1,2),(1,3),(2,3)\}, \{(1,2),(2,2),(2,3)\}, \\ & \{(1,3),(2,2),(2,3)\}, \{(1,2),(1,3),(2,2),(2,3)\}\}.\end{aligned}$$

Q2 a) $\mathcal{P}(\emptyset) = \{\emptyset\}$.

b) $\mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$, so $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$.

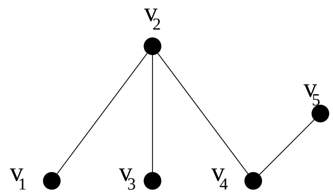
Exercise Set 11.1

Q4



Q9 i) e_1, e_2, e_7 ii) v_1, v_2 iii) e_2, e_7 iv) e_1, e_3, v) e_4, e_5 vi) v_4 vii) $deg(v_3) = 2$
viii) total degree is 14.

Q21



Q23 Let v represent the number of vertices and since the sum of the degrees of the vertices must equal twice the number of edges, we have $3v = 18$. So $v = 6$. Now the corresponding graph is: