MATH1061/7861 Semester 2, 2005 Solution Ass. 9

Section G.1: Question 1

(i) Element a is the identity of $(H, +)$ and element A is the identity of $(K, \star)$.

(ii)

$$\begin{aligned} (B * B) * C &= E * C = B \\ B * (B * C) &= B * D = A \end{aligned}$$

Since $(B * B) * C \neq B * (B * C)$, $(K, \star)$ is not associative, and hence not a group.

(iii) Yes, element d.

(iv) Define $f : \{0, 1, 2, 3, 4\} \to \{a, b, c, d, e\}$ and $f(0) = a,\ f(1) = d,\ f(2) = b,\ f(3) = c, f(4) = e$.

Exercise Set 6.1

Q14 a) $3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$.

b) The probability that exactly two people become ill is $3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$. The probability that all three people become ill is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. So the answer is $\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$.

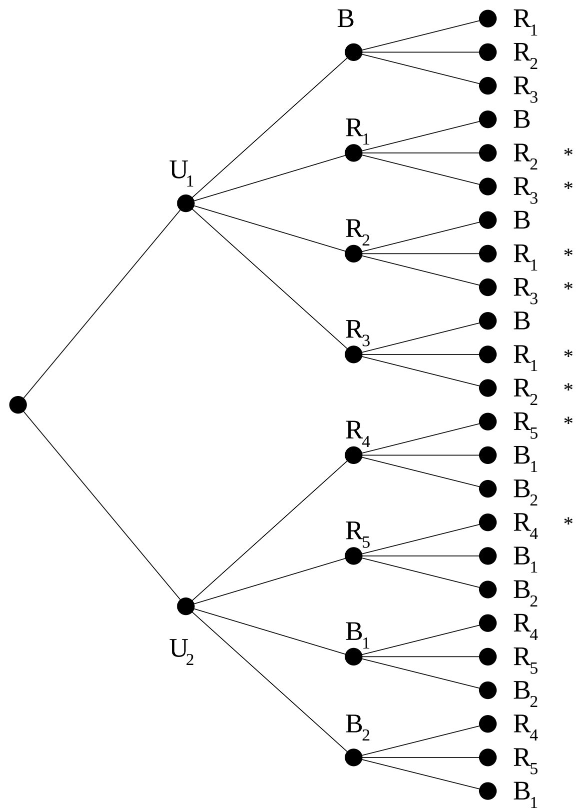
c) $\frac{1}{8}$.

Q32 a) $\frac{365}{7} = 52\frac{1}{7}$. Hence 52 Sundays.

b) Since there are $52\frac{1}{7}$ weeks there will be 53 Mondays.

Exercise Set 6.2

Q7 a)



b) $1 \times 4 \times 3 + 1 \times 4 \times 3 = 2 \times 4 \times 3 = 24$.

c) From above those branches marked with an *, give two red balls. Hence probability is $\frac{8}{24} = \frac{1}{3}$.

Exercise Set 6.3

Q16 b) Let S denote the sample space and $A, B \subseteq S$. Then

$$\begin{aligned} LHS &= P(A \cup B) = \frac{n(A \cup B)}{n(S)} \\ RHS &= P(A) + P(B) - P(A \cap B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}. \end{aligned}$$

And since $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$\begin{aligned} RHS &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A) + n(B) - n(A \cup B)}{n(S)} \\ &= \frac{n(A \cup B)}{n(S)}. \end{aligned}$$

Therefore $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ as required.

Q20 a) Total number of n bit strings is 4^n. Number of strings with all consecutive bits different is $4.3.3 \ldots 3 = 4.3^{n-1}$. So the number of strings with at least two consecutive bits the same is $4^n - 4.3^{n-1}$.

b) When $n = 10$ we obtain $\dfrac{4^{10} - 4.3^9}{4^{10}} = 1 - (\frac{3}{4})^9 \approx 92.5\%$.