DEPARTMENT OF MATHEMATICS

MATH3301 GRAPH THEORY

ASSIGNMENT ONE

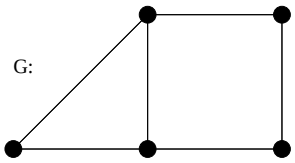
DUE DATE for ASSIGNMENT ONE: 3pm TUESDAY 8th August 2006

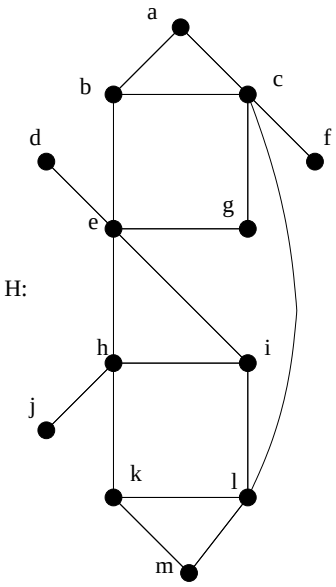
Q1 Determine whether the following sequences are graphical; if one is, then draw a simple graph with that degree sequence; if not, explain briefly why not.

(i) $S_1:\ 4,4,3,3,2,2.$

(ii) $S_2:\ 8,6,6,5,4,3,2,1,1.$

Q2 Find the number $\tau(G)$ of spanning trees for the graph G shown on the left below.





Q3 Solve the Chinese Postman Problem for the given graph H shown on the right above. Ensure you give all steps in the algorithm. Also if it is necessary to add any edges, then redraw the graph and explain why the extended graph is eulerian. Name an eulerian cycle.

Q4 Recall that the product of two graphs G_1 and G_2 is defined to be $G_1 \times G_2$, where $V(G_1 \times G_2) = V_1 \times V_2$ and, the edges set $E(G_1 \times G_2)$ is defined as the set of edges

$$\{(x_1,x_2),(y_1,y_2)\} \text{ where } \begin{cases} x_1 = y_1 \text{ and } \{x_2,y_2\} \in E(G_2), \text{ or} \\ x_2 = y_2 \text{ and } \{x_1,y_1\} \in E(G_1) \end{cases}$$

(i) Draw the graphs, K_2 and K_3 and label the vertices.

(ii) Write out explicitly the vertex set and edge set of the graph $K_2 \times K_3$. (Note the elements of the vertex set should be ordered pairs with coordinates based on the labeling you chose in part (i)).

(iii) Draw the graph $K_2 \times K_3$. Draw the graph $K_3 \times K_3$

(iv) Is $K_2 \times K_3$ Hamiltonian? Is $K_3 \times K_3$ Hamiltonian? If so draw such a cycle.