DEPARTMENT OF MATHEMATICS

MATH3301 GRAPH THEORY

ASSIGNMENT THREE

DUE DATE for ASSIGNMENT THREE: 3pm TUESDAY 5th of September 2006

Q1 Prove that for every positive integer n,

$$r(3,n) \leq \frac{n^2+n}{2}.$$

Q2 Show that (i) $r(5,5) \leq 50$, and (ii) $r(4,6) \leq 43$.
Hint: Use the list of ramsey numbers on slides 14 and 19 given in notes.

Q3 Let $m \geq 2$ and $n \geq 2$ be integers, and let $p = r(m,n) - 1$. Suppose that each edge of K_p is arbitrarily coloured red or blue. Show that:

(i) K_p contains a red K_{m-1} or a blue K_n.

(ii) K_p contains a red K_m or a blue K_{n-1}.

Q4 Prove that K_n has a 2-factorization if and only if n is odd.

Q5 Let G be a graph on $2m$ vertices, with $\delta(G) = m+1$ ($m \geq 1$). Show that G must contain a triangle.