MATH3301 GRAPH THEORY

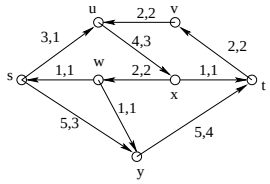
ASSIGNMENT 4

DUE DATE for ASSIGNMENT FOUR: 5pm TUESDAY 3rd October 2006

Questions 1, 2, 6, 7, 9 and 10 from Sheet 6 given below.

SHEET 6

1. Shown below is a network N and a flow f_1. Each arc is labelled with its capacity and the flow along the arc.



(i) What is the value of $f_1(N)$?

(ii) Find a maximum flow f for N. Explain why f must be a maximum flow. What is a minimum cut in N?

2. Let $\mathcal{P} = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{L} = \{\{1, 2, 4\}, \{1, 3\}, \{2, 3\}, \{3, 4, 5\}, \{1, 5\}, \{2, 5\}\}$ Draw a diagram of the incidence structure $(\mathcal{P}, \mathcal{L})$. Is $(\mathcal{P}, \mathcal{L})$ a near–linear space? Is $(\mathcal{P}, \mathcal{L})$ a linear space?

3. Let $\mathcal{P} = \{1, 2, 3, 4, 5, 6, 7\}$ and $\mathcal{L} = \{\{7, 1, 3\}, \{6, 7, 2\}, \{3, 4, 6\}, \{2, 3, 5\}, \{4, 5, 7\}, \{5, 6, 1\}, \{1, 2, 4\}\}$. Draw a diagram of this incidence structure. Is $S = (\mathcal{P}, \mathcal{L})$ a near–linear space? Is $(\mathcal{P}, \mathcal{L})$ a linear space?

4. Let $(\mathcal{P}, \mathcal{L})$ be a near–linear space. Prove that if L and M are lines of $\mathcal{L}$ such that for each point P incident with L the point P is also incident with M, then $M = L$.

5. Let $S = (\mathcal{P}, \mathcal{L})$ be a near–linear space, the **line graph** $R = (\mathcal{P}', \mathcal{L}')$ of S is defined by

- $\mathcal{P}' = \mathcal{L}$
- $\mathcal{L}' = \{\{L, M\} \mid \text{where } L \text{ and } M \text{ are distinct intersecting lines of } \mathcal{L}\}$

Draw the line graph of the near linear space $S = (\mathcal{P}, \mathcal{L})$ given in Question 2 above.

6. Find a basis for the near–linear space given in Question 3 above. Explain your answer.

7. Let $(\mathcal{X}, \mathcal{L}_{\mathcal{X}})$ be a subspace of the near-linear space given in Question 3 above. Assuming $|\mathcal{X}| \geq 1$ and $|\mathcal{L}_{\mathcal{X}}| \geq 1$. What are the possible sizes for $|\mathcal{X}|$.

8. Given $S = (\mathcal{P}, \mathcal{L})$, where $\mathcal{P} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\mathcal{L} = \{\{1, 2, 8\}, \{1, 3, 9\}, \{2, 4, 6, 9\}, \{3, 5, 7, 8\}, \{2, 3\}, \{4, 5\}, \{4, 7\}, \{5, 6\}, \{6, 7\}\}$. Find a basis for S which contains 3 points and a basis which contains 4 points. What is the dimension of S? Justify your answer.

9. Draw a diagram for the incidence structure $\mathcal{I} = (\mathcal{P}, \mathcal{L})$, where $\mathcal{P} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $\mathcal{L} = \{\{1, 2, 3\}, \{1, 4, 7\}, \{2, 4, 6\}, \{3, 4, 5\}, \{4, 8\}, \{5, 6, 7\}\}$.

(i) Is this incidence structure a near-linear space?

(ii) Is the set $\{1, 2, 7\}$ a subspace?

(iii) If not find the subspace generated by this set. That is find $< \{1, 2, 7\} >$.

(iv) Find a basis for the incidence structure $\mathcal{I} = (\mathcal{P}, \mathcal{L})$.

(v) What is the dimension of the incidence structure $\mathcal{I} = (\mathcal{P}, \mathcal{L})$.

10. Adapt (by adding new lines or additional points to existing lines) so that the near-linear space given in Question 9 above is a linear space. Draw a diagram of this incidence structure.

11. Use the complete graph K_4 to define an incidence structure $(\mathcal{P}, \mathcal{L})$. Is this incidence structure a linear space? Does it have point and line regularity? If so give the point and line regularity.