MATH3301 GEOMETRY

ASSIGNMENT 5

DUE DATE for ASSIGNMENT FOUR: 5pm TUESDAY 17TH October 2006

Questions 1, 3, 6, 9 and 11 from Sheet 7 given below.

SHEET 7

1. Let a linear space on v points and b lines have line and point regularity $k+1$, $k \geq 1$. Prove that all lines meet and $b = v = k^2 + k + 1$.

2. Take the Euclidean plane $\mathbb{R}^2$. Let a point of $\mathbb{R}^2$ be an ordered pair (x, y) and a line be the set of points (x, y) which satisfy an equation $y = ax + b$ or $x = c$, where a, b and c are real numbers.

   i. Is $\mathbb{R}^2$ a linear space?

   ii. Let $\mathcal{P}' = \{(x, y) \mid x^2 + y^2 < 1\}$. (Note $\mathcal{P}'$ is the set of points inside the unit circle.) Let $\mathcal{L}'$ be the set of restrictions of lines of $\mathbb{R}^2$ to $\mathcal{P}'$. Is $(\mathcal{P}', \mathcal{L}')$ a linear space?

3. Let (v, b, r, k, λ) be the parameters of a balanced incomplete block design, where $k = 3$ and $\lambda = 1$. Prove that

$$v \equiv 1, 3 (\text{mod } 6).$$

4. State the dual of the following result.

   Let π be a finite projective plane. Every pair of distinct lines of π meet at exactly one point of π.

5. Write out the lines of the dual of the projective plane $\pi = (\mathcal{P}, \mathcal{L})$ where $\mathcal{P} = \{1, 2, 3, 4, 5, 6, 7\}$ and $\mathcal{L} = \{\{1, 2, 3\}, \{1, 4, 5\}, \{1, 6, 7\}, \{2, 4, 6\}, \{2, 5, 7\}, \{3, 4, 7\}, \{3, 5, 6\}\}$. Is the dual a projective plane?

6. Let $\mathcal{Q} = \{P_1, P_2, P_3, P_4\}$ be a quadrangle of π, a projective plane of order 2. Let L be a line in π. Can $|L \cap \mathcal{Q}| = 1$?

7. Define the terms

   (i) $\overline{\mathcal{L}}$–secant,

   (ii) $\overline{\mathcal{L}}$-tangent, and

   (iii) $\overline{\mathcal{L}}$-exterior point,

where $\overline{\mathcal{L}}$ is a set of lines of a finite projective plane.

8. Consider the projective plane given in Question 5 above. Partition the lines of π into

   (i) $\overline{\mathcal{P}}$–secants,

   (ii) $\overline{\mathcal{P}}$-tangents, and

(iii) $\overline{\mathcal{P}}$-exterior lines,

where

(a) $\overline{\mathcal{P}} = \{1\}$,

(b) $\overline{\mathcal{P}} = \{1, 2\}$,

(c) $\overline{\mathcal{P}} = \{1, 2, 3\}$,

(d) $\overline{\mathcal{P}} = \{1, 2, 3, 6\}$.

9.(i) How many points does a projective plane of order 9 have?

(ii) How many lines does a projective plane of order 9 have?

(iii) How many lines are through a point of a projective plane of order 5?

10. A projective plane π has 273 points.

(i) What is the order of π?

(ii) How many points are there on a line of π?

11. Let $\mathcal{Q} = \{P_1, P_2, P_3, P_4\}$ be a quadrangle of a projective plane π of order four, $\mathcal{D} = \{D_1, D_2, D_3\}$ be the set of diagonal points of $\mathcal{Q}$ and $\overline{\mathcal{Q}} = \mathcal{Q} \cup \mathcal{D}$. In π there are six $\mathcal{Q}$–secants (namely $P_1P_2, P_1P_3, P_1P_4, P_2P_3, P_2P_4, P_3P_4$). Suppose that D_1, D_2 and D_3 are not collinear. Then there are three $\mathcal{D}$–secants (namely D_1D_2, D_1D_3, D_2D_3). The $\mathcal{Q}$–secants and $\mathcal{D}$–secants are all of the $\overline{\mathcal{Q}}$–secants.

(a) Retaining the supposition that D_1, D_2 and D_3 are not collinear, answer the following questions:

(i) How many $\overline{\mathcal{Q}}$–secants are there on each point of $\mathcal{Q}$?

(ii) How many $\overline{\mathcal{Q}}$–tangents are there on each point of $\mathcal{Q}$?

(iii) How many $\overline{\mathcal{Q}}$–secants are there on each point of $\mathcal{D}$?

(iv) How many $\overline{\mathcal{Q}}$–tangents are there on each point of $\mathcal{D}$?

(v) How many $\overline{\mathcal{Q}}$–tangents are there in all?

(b) (i) Show that, if the diagonal points D_1, D_2 and D_3 are not collinear, then there is precisely one $\overline{\mathcal{Q}}$–exterior line.

(ii) Let L be a $\overline{\mathcal{Q}}$–exterior line. (Such a line exists by (b)(i) if the diagonal points D_1, D_2 and D_3 are not collinear.) Show that $|L| \geq 6$. Deduce that the diagonal points of a quadrangle of a projective plane of order four are collinear.