**MATHEMATICS DEPARTMENT: MATH3301 GEOMETRY**

MATH3301: Problem Sheet 8

1. Let $\pi_1 = (\mathcal{P}_1, \mathcal{L}_1)$ and $\pi_2 = (\mathcal{P}_2, \mathcal{L}_2)$ be two projective planes of order $n$ and $\phi$ be an isomorphism from $\pi_1$ onto $\pi_2$. Prove that

(i) $\phi$ is one–to–one from $\mathcal{P}_1$ onto $\mathcal{P}_2$ and

(ii) $\phi$ is one–to–one from $\mathcal{L}_1$ onto $\mathcal{L}_2$.

2. Let $\pi_1 = (\mathcal{P}_1, \mathcal{L}_1)$ and $\pi_2 = (\mathcal{P}_2, \mathcal{L}_2)$ be two projective planes of order $n$ and $\phi$ be an isomorphism from $\pi_1$ onto $\pi_2$. Prove that $\phi$ preserves non–incidence. (That is, $\phi$ is such that, if $P_1$ is not on $L_1$ in $\pi_1$, then $\phi(P_1)$ is not on $\phi(L_1)$ in $\pi_2$, or equivalently if $\phi(P_1)$ is on $\phi(L_1)$, then $P_1$ is on $L_1$.)

3. Let $\mathcal{Q}$ be a quadrangle of a finite projective plane $\pi$. Show that, if the diagonal points $D_1$, $D_2$ and $D_3$ are collinear (on a line $L_\infty$ say), then the substructure of $\pi$ defined obtained from the restriction of $\pi$ to the set of points

$$\mathcal{Q} \cup \{D_1, D_2, D_3\}$$

is a projective plane of order two.

4. A collection $C$ of axioms is said to be *self–dual* if, for each axiom $A$ in $C$, the dual of $A$ is in $C$ or follows from $C$.

Show that the collection of axioms $A_1$, $A_5$, $A_6$ and $A_7$ are not self–dual.

5. Show that any finite affine plane possesses a set of four points no three of which are collinear.

6. Let $\alpha$ be an affine plane of order $n$ and let $L$ be a line of $\alpha$. Show that the number of lines meeting $L$ in a unique point is $n^2$.

7. Let $\alpha = (\mathcal{P}_\alpha, \mathcal{L}_\alpha)$ be an affine plane of order 3.

(i) What is $|\mathcal{P}_\alpha|$?

(ii) What is $|\mathcal{L}_\alpha|$?

(iii) What is $|P|$, where $P \in \mathcal{P}_\alpha$?

(iv) What is $|L|$, where $L \in \mathcal{L}_\alpha$?

(v) What is $|c|$, where $c$ is a parallel class of $\alpha$?

(vi) How many parallel classes are there in $\alpha$?

(vii) Construct an affine plane of order 3.

(viii) Find the projective completion of the plane constructed in (vii).