1. Let $\pi_1 = (P_1, L_1)$ and $\pi_2 = (P_2, L_2)$ be two projective planes of order $n$ and $\phi$ be an isomorphism from $\pi_1$ onto $\pi_2$. Prove that

(i) $\phi$ is one–to–one from $P_1$ onto $P_2$ and
(ii) $\phi$ is one–to–one from $L_1$ onto $L_2$.

2. Let $\pi_1 = (P_1, L_1)$ and $\pi_2 = (P_2, L_2)$ be two projective planes of order $n$ and $\phi$ be an isomorphism from $\pi_1$ onto $\pi_2$. Prove that $\phi$ preserves non–incidence. (That is, $\phi$ is such that, if $P_1$ is not on $L_1$ in $\pi_1$, then $\phi(P_1)$ is not on $\phi(L_1)$ in $\pi_2$, or equivalently if $\phi(P_1)$ is on $\phi(L_1)$, then $P_1$ is on $L_1$.)

3. Let $Q$ be a quadrangle of a finite projective plane $\pi$. Show that, if the diagonal points $D_1$, $D_2$ and $D_3$ are collinear (on a line $L_\infty$ say), then the substructure of $\pi$ defined obtained from the restriction of $\pi$ to the set of points

$$Q \cup \{D_1, D_2, D_3\}$$

is a projective plane of order two.

4. A collection $C$ of axioms is said to be self–dual if, for each axiom $A$ in $C$, the dual of $A$ is in $C$ or follows from $C$.
   Show that the collection of axioms $A_1$, $A_5$, $A_6$ and $A_7$ are not self–dual.

5. Show that any finite affine plane possesses a set of four points no three of which are collinear.

6. Let $\alpha$ be an affine plane of order $n$ and let $L$ be a line of $\alpha$. Show that the number of lines meeting $L$ in a unique point is $n^2$.

7. Let $\alpha = (P_\alpha, L_\alpha)$ be an affine plane of order 3.

   (i) What is $|P_\alpha|$?
   (ii) What is $|L_\alpha|$?
   (iii) What is $|P|$, where $P \in P_\alpha$?
   (iv) What is $|L|$, where $L \in L_\alpha$?
   (v) What is $|c|$, where $c$ is a parallel class of $\alpha$?
   (vi) How many parallel classes are there in $\alpha$?
   (vii) Construct an affine plane of order 3.
   (viii) Find the projective completion of the plane constructed in (vii).