MATHEMATICS DEPARTMENT: MATH3301 GEOMETRY

Problem Sheet 9

1. Find all the "trivial" finite affine planes; that is find all incidence structures which satisfy axioms A1 and A5 but not A6.

2. Let ϕ be an isomorphism from the affine plane $\alpha_1 = (\mathcal{P}_1, \mathcal{L}_1)$ onto the affine plane $\alpha_2 = (\mathcal{P}_2, \mathcal{L}_2)$, where they are both of order n. Show that

(i) ϕ is one–to–one on the set of points and on the set of lines,

(ii) if $\phi(P_1)$ is on $\phi(L_1)$ then P_1 is on L_1, and

(iii) ϕ^{-1} is an isomorphism from α_1 to α_2.

3. Let ϕ be a collineation of a finite affine plane α. Show that if ϕ fixes a pair of distinct points P and Q of α then ϕ fixes the line PQ of α.

4. State and prove an analogous result to Question 3 for a pair of distinct lines L and M of α.

5. Show that, if π_0 is a projective subplane of π, then π_0^d, the dual of π_0, is a projective subplane of π^d the dual of π.

6.(i) Find an affine subplane of order two of the projective plane of order three constructed in Question 7 Problem Sheet 4.

(ii) Verify that the set of exterior points of the line set of your affine subplane is empty and that the set of tangent points of the line set of your affine subplane is not empty.

7. Let π_0 be a projective subplane, of order m, of a projective plane π of order n, where $m < n$. Prove that

$$n = m^2 \quad \text{or} \quad n \geq m^2 + m.$$

8. What is the order of a Baer subplane of a projective plane of order four?

9. Show that any quadrangle of a projective plane π of order four is contained in a Bear subplane of π.