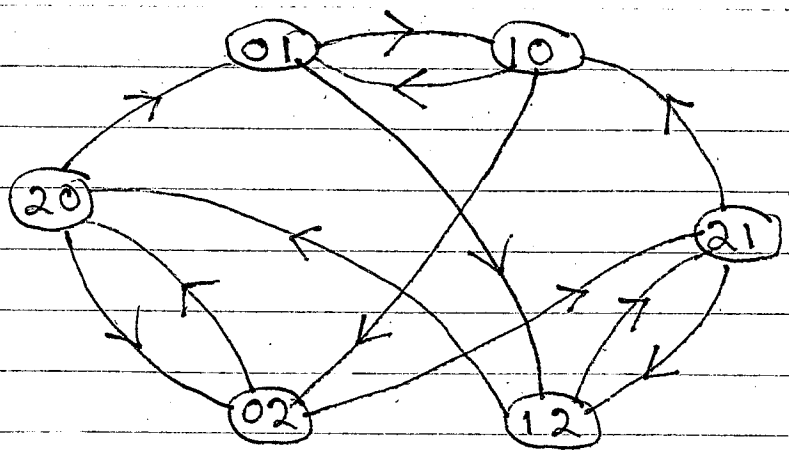
Solutions Assignment 2

Q1 i) Vertex set $V = \{01, 10, 02, 20, 12, 21\}$

Edge labels. $\{010, 012, 101, 102, 210, 212, 121, 120, 021, 020, 201, 202\}$.

giving the following digraph



ii) 201012102120 20

Q2. a) Let the vertex set of $K_{n,n+1}$ be $V_1 \cup V_2$ where $V_1 = \{v_1, \ldots, v_n\}$ and $V_2 = \{u_1, \ldots, u_{n+1}\}$.

Delete the set of vertices V_1. Then the remaining graph is made up of $n+1$ isolated vertices. Hence the number of components is $n+1 > |V_1|$. Thus no hamiltonian cycle exists

Solutions Assignment 2

b) Let G be a bipartite graph of odd order with vertex set $V(G) = V_1 \cup U_2$. Then without loss of generality $|V_1| = 2m+1$ and $|U_2| = 2n$ for positive integers m, n. If $|V_1| > |U_2|$ delete U_2 from the vertex set of G and what remains is $|V_1|$ isolated vertices. Hence the number of remaining component will be greater than $|U_2|$. Thus no hamiltonian cycle exists.

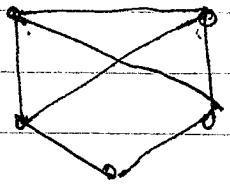
If $|U_2| > |V_1|$ we repeat the argument, but this time removing V_1 from the set of vertices.

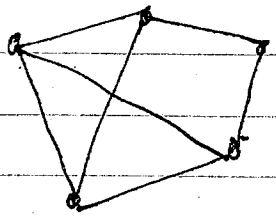
Q3. We must identify a subset, S, of vertices which when removed give more components that $|S|$. So let $S = \{u, a, y, w\}$. Then $|S| = 4$

Solutions Assignment 2

MATH 3301 (2006) (3)

G − S

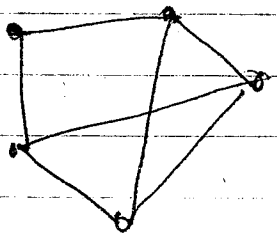




z

v

x

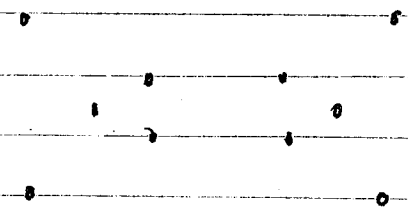


$k(G-S) = 6 > |S| = 4$.

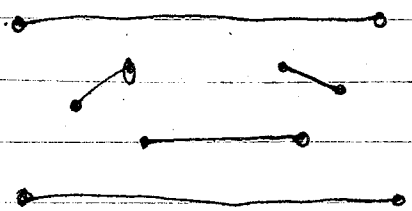
Thus by Tutte's theorem no 1-factor exists.

Q 4.

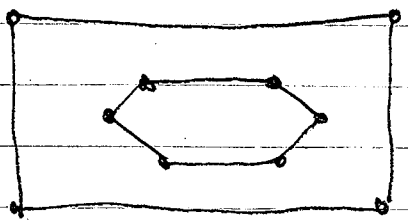
$k=0$



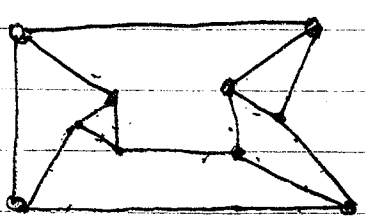
$k=1$



$k=2$

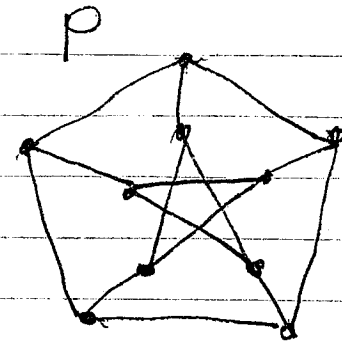


$k=3$



Solutions Assignment 2

Q 5. The Petersen graph is



Assume it contains a hamiltonian cycle C.

Since $|V(P)| = 10$

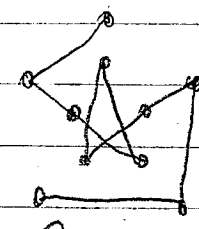
the number of edges in C is 10.

If we take alternating edges in C we obtain 2 one factors in P.

Since P is 3 regular. $E(P) \setminus E(C)$ must also form a 1-factor of P.

Hence the edges of P can be partitioned into 3 1-factors. However this would imply P contains a 1-factorisation, which it does not. Thus we have a contradiction and so P does not contain a hamiltonian cycle.

But it does contain a hamiltonian path,



for instance.