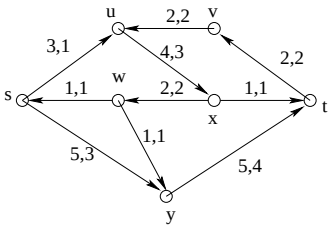
MATH3301: Solutions to Problem Sheet 6

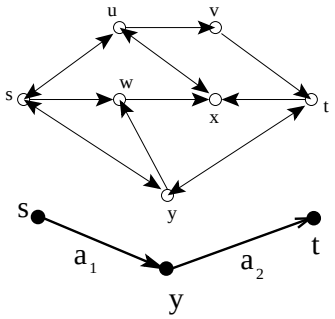
1. (i)

$$\sum_{y\in N^-(t)} f(y,t) - \sum_{y\in N^+(t)} f(t,y) = 1+4-2 = 3.$$

(ii)



network digraph D

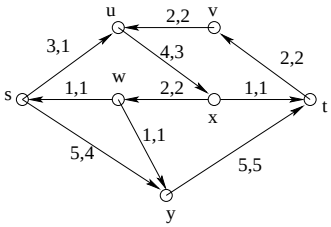


digraph D'

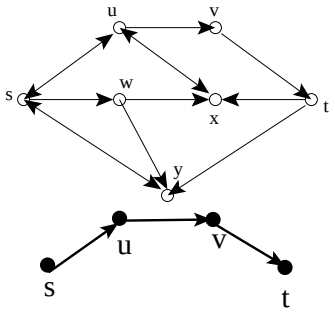
The shortest path from s to t is displayed above, with associated Δs:

$$\Delta_1 = 5-3 = 2, \quad \Delta_2 = 5-4 = 1, \text{ so } \Delta = 1$$

Now updating the network and constructing a new digraph D' gives



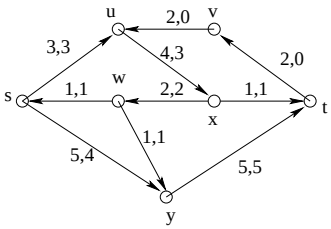
network digraph D



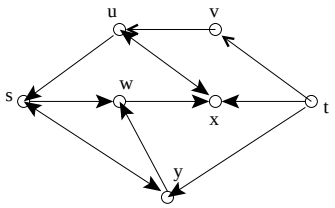
digraph D'

The shortest path from s to t is displayed above, with associated Δs:

$$\Delta_1 = 3-1 = 2, \quad \Delta_2 = 2, \quad \Delta_3 = 2, \text{ so } \Delta = 2$$



network digraph D

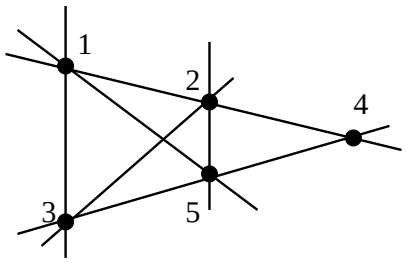


digraph D'

Now there are no paths in D' from s to t. Thus a minimum cut will be given by $P = \{s,u,w,y,x\}$ and $\overline{P} = \{v,t\}$ and we have

$$\sum_{(x,y)\in c(P,\overline{P})} c(x,y) - \sum_{(x,y)\in c(\overline{P},P)} c(x,y) = \sum_{y\in N^-(t)} f(y,t) - \sum_{y\in N^+(t)} f(t,y) = 5+1-0 = 6.$$

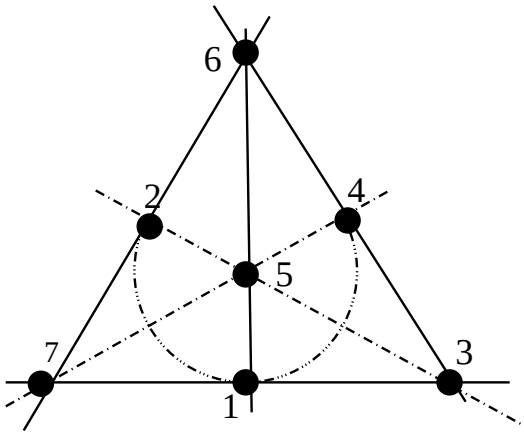
2.



Every line is on at least two points hence Axiom NL1 is satisfied. Any two points are on at most one line so Axiom NL2 is satisfied. Therefore it is a near–linear space.

No as every pair of points is not joined by a line, consider point 6. Hence $(\mathcal{P}, \mathcal{L})$ is not a linear space.

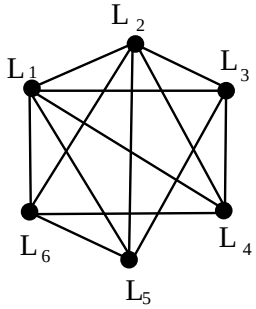
3.



Yes it is as it satisfies Axioms NL1 and NL2. In addition, every pair of points is joined by a line, hence $(\mathcal{P}, \mathcal{L})$ is also a linear space.

4. By Axiom NL1 L is on at least two points, and any two points are on at most one line. Hence $M = L$.

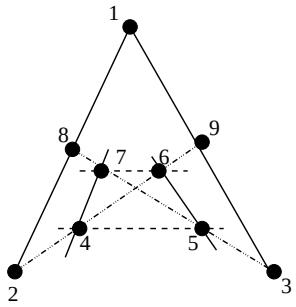
5. Let $L_1 = \{1, 2, 4\}$, $L_2 = \{3, 4, 5\}$, $L_3 = \{1, 3\}$, $L_4 = \{2, 3\}$, $L_5 = \{1, 5\}$ and $L_6 = \{2, 5\}$. So $\mathcal{P}' = \{L_1, L_2, L_3, L_4, L_5, L_6\}$ and $\mathcal{L}' = \{\{L_1, L_2\}, \{L_1, L_3\}, \{L_1, L_4\}, \{L_1, L_5\}, \{L_1, L_6\}, \{L_2, L_3\}, \{L_2, L_4\}, \{L_2, L_5\}, \{L_2, L_6\}, \{L_4, L_6\}, \{L_5, L_6\}\}$



6. The set $\{2, 3, 7\}$ generates the space, because these three points generate the entire space, while any two points will only generate a line.

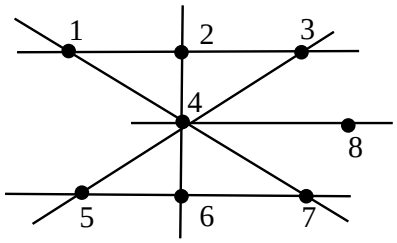
7. The answer is $|\mathcal{X}| = 3$ or 7. Since $|\mathcal{L}_{\mathcal{X}}| \geq 1$, we know that $|\mathcal{X}| \geq 3$, and any line will be a subspace hence it is possible for $|\mathcal{X}| = 3$. Now if we take $\mathcal{X}$ to contain 3 points not on a line, then they form a basis and generate the entire space. In this case $|\mathcal{X}| = 7$.

8.



The sets $\{1, 2, 3\}$ and $\{4, 5, 6, 7\}$ are bases for S. To check that this is infact a basis you must check that it generates the whole space and that if you remove an element from either of these sets then you do not get the entire space. Note there is not a set of two points which generates the whole space as any two points will only generate a line. However $\{1, 2, 3\}$ generates the entire space so the dimension of this near–linear space is 2.

9.



(i) Yes, every line contains at least two points and every pair of points is on at most one line.

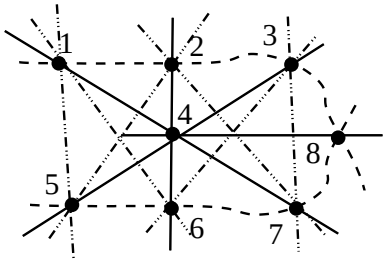
(ii) No, since the point 4 is on the line incident with points 1 and 7, but the point 4 is not a member of the set $\{1, 2, 7\}$.

(iii) $\langle\{1, 2, 7\}\rangle = (\chi, \mathcal{L})$, where $\chi = \{1, 2, 3, 4, 5, 6, 7\}$ and $\mathcal{L} = \{\{1, 2, 3\}, \{1, 4, 7\}, \{2, 4, 6\}, \{3, 4, 5\}, \{5, 6, 7\}\}$.

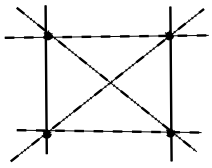
(iv) $\{1, 2, 4, 8\}$

(v) The dimension for the entire space is $4 - 1 = 3$.

10. One possibility is, the linear space is $S = (\mathcal{P}, \mathcal{L}')$, where $\mathcal{P} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $\mathcal{L}' = \{\{1, 2, 3, 8\}, \{1, 4, 7\}, \{1, 5\}, \{1, 6\}, \{2, 4, 6\}, \{2, 5\}, \{2, 7\}, \{3, 4, 5\}, \{3, 7\}, \{3, 6\}, \{4, 8\}, \{5, 6, 7, 8\}\}$.



11. K_4 is given by:



Yes it is a linear space because each line is on precisely two points and each pair of points is on precisely one line. Yes it is line and point regular. The line regularity is 2 and the point regularity is 3.