CONTENTS

Preface xvii
Acknowledgments xix

1 Uniform Random Number Generation 1
  1.1 Random Numbers 1
     1.1.1 Properties of a Good Random Number Generator 2
     1.1.2 Choosing a Good Random Number Generator 3
  1.2 Generators Based on Linear Recurrences 4
     1.2.1 Linear Congruential Generators 4
     1.2.2 Multiple-Recursive Generators 5
     1.2.3 Matrix Congruential Generators 6
     1.2.4 Modulo 2 Linear Generators 6
  1.3 Combined Generators 8
  1.4 Other Generators 10
  1.5 Tests for Random Number Generators 11
     1.5.1 Spectral Test 12
     1.5.2 Empirical Tests 14
  References 21
CONTENTS

2 Quasirandom Number Generation 25
  2.1 Multidimensional Integration 25
  2.2 Van der Corput and Digital Sequences 27
  2.3 Halton Sequences 29
  2.4 Faure Sequences 31
  2.5 Sobol’ Sequences 33
  2.6 Lattice Methods 36
  2.7 Randomization and Scrambling 38
    References 40

3 Random Variable Generation 43
  3.1 Generic Algorithms Based on Common Transformations 44
    3.1.1 Inverse-Transform Method 45
    3.1.2 Other Transformation Methods 47
    3.1.3 Table Lookup Method 55
    3.1.4 Alias Method 56
    3.1.5 Acceptance–Rejection Method 59
    3.1.6 Ratio of Uniforms Method 66
  3.2 Generation Methods for Multivariate Random Variables 67
    3.2.1 Copulas 68
  3.3 Generation Methods for Various Random Objects 70
    3.3.1 Generating Order Statistics 70
    3.3.2 Generating Uniform Random Vectors in a Simplex 71
    3.3.3 Generating Random Vectors Uniformly Distributed in
         a Unit Hyperball and Hypersphere 74
    3.3.4 Generating Random Vectors Uniformly Distributed in
         a Hyperellipsoid 75
    3.3.5 Uniform Sampling on a Curve 75
    3.3.6 Uniform Sampling on a Surface 76
    3.3.7 Generating Random Permutations 79
    3.3.8 Exact Sampling From a Conditional Bernoulli
         Distribution 80
    References 83

4 Probability Distributions 85
  4.1 Discrete Distributions 85
    4.1.1 Bernoulli Distribution 85
    4.1.2 Binomial Distribution 86
    4.1.3 Geometric Distribution 91
    4.1.4 Hypergeometric Distribution 93
    4.1.5 Negative Binomial Distribution 94
<table>
<thead>
<tr>
<th>4.1.6</th>
<th>Phase-Type Distribution (Discrete Case)</th>
<th>96</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.7</td>
<td>Poisson Distribution</td>
<td>98</td>
</tr>
<tr>
<td>4.1.8</td>
<td>Uniform Distribution (Discrete Case)</td>
<td>101</td>
</tr>
<tr>
<td>4.2</td>
<td>Continuous Distributions</td>
<td>102</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Beta Distribution</td>
<td>102</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Cauchy Distribution</td>
<td>106</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Exponential Distribution</td>
<td>108</td>
</tr>
<tr>
<td>4.2.4</td>
<td>F Distribution</td>
<td>109</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Fréchet Distribution</td>
<td>111</td>
</tr>
<tr>
<td>4.2.6</td>
<td>Gamma Distribution</td>
<td>112</td>
</tr>
<tr>
<td>4.2.7</td>
<td>Gumbel Distribution</td>
<td>116</td>
</tr>
<tr>
<td>4.2.8</td>
<td>Laplace Distribution</td>
<td>118</td>
</tr>
<tr>
<td>4.2.9</td>
<td>Logistic Distribution</td>
<td>119</td>
</tr>
<tr>
<td>4.2.10</td>
<td>Log-Normal Distribution</td>
<td>120</td>
</tr>
<tr>
<td>4.2.11</td>
<td>Normal Distribution</td>
<td>122</td>
</tr>
<tr>
<td>4.2.12</td>
<td>Pareto Distribution</td>
<td>125</td>
</tr>
<tr>
<td>4.2.13</td>
<td>Phase-Type Distribution (Continuous Case)</td>
<td>126</td>
</tr>
<tr>
<td>4.2.14</td>
<td>Stable Distribution</td>
<td>129</td>
</tr>
<tr>
<td>4.2.15</td>
<td>Student’s t Distribution</td>
<td>131</td>
</tr>
<tr>
<td>4.2.16</td>
<td>Uniform Distribution (Continuous Case)</td>
<td>134</td>
</tr>
<tr>
<td>4.2.17</td>
<td>Wald Distribution</td>
<td>135</td>
</tr>
<tr>
<td>4.2.18</td>
<td>Weibull Distribution</td>
<td>137</td>
</tr>
<tr>
<td>4.3</td>
<td>Multivariate Distributions</td>
<td>138</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Dirichlet Distribution</td>
<td>139</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Multinomial Distribution</td>
<td>141</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Multivariate Normal Distribution</td>
<td>143</td>
</tr>
<tr>
<td>4.3.4</td>
<td>Multivariate Student’s t Distribution</td>
<td>147</td>
</tr>
<tr>
<td>4.3.5</td>
<td>Wishart Distribution</td>
<td>148</td>
</tr>
<tr>
<td>4.3</td>
<td>References</td>
<td>150</td>
</tr>
</tbody>
</table>

5 Random Process Generation 153

5.1 Gaussian Processes 154

5.1.1 Markovian Gaussian Processes 159

5.1.2 Stationary Gaussian Processes and the FFT 160

5.2 Markov Chains 162

5.3 Markov Jump Processes 166

5.4 Poisson Processes 170

5.4.1 Compound Poisson Process 174

5.5 Wiener Process and Brownian Motion 177

5.6 Stochastic Differential Equations and Diffusion Processes 183

5.6.1 Euler’s Method 185

5.6.2 Milstein’s Method 187
8 Statistical Analysis of Simulation Data 301
  8.1 Simulation Data 301
    8.1.1 Data Visualization 302
    8.1.2 Data Summarization 303
  8.2 Estimation of Performance Measures for Independent Data 305
    8.2.1 Delta Method 308
  8.3 Estimation of Steady-State Performance Measures 309
    8.3.1 Covariance Method 309
    8.3.2 Batch Means Method 311
    8.3.3 Regenerative Method 313
  8.4 Empirical Cdf 316
  8.5 Kernel Density Estimation 319
    8.5.1 Least Squares Cross Validation 321
    8.5.2 Plug-in Bandwidth Selection 326
  8.6 Resampling and the Bootstrap Method 331
  8.7 Goodness of Fit 333
    8.7.1 Graphical Procedures 334
    8.7.2 Kolmogorov–Smirnov Test 336
    8.7.3 Anderson–Darling Test 339
    8.7.4 $\chi^2$ Tests 340
  References 343

9 Variance Reduction 347
  9.1 Variance Reduction Example 348
  9.2 Antithetic Random Variables 349
  9.3 Control Variables 351
  9.4 Conditional Monte Carlo 354
  9.5 Stratified Sampling 356
  9.6 Latin Hypercube Sampling 360
  9.7 Importance Sampling 362
    9.7.1 Minimum-Variance Density 363
    9.7.2 Variance Minimization Method 364
    9.7.3 Cross-Entropy Method 366
    9.7.4 Weighted Importance Sampling 368
    9.7.5 Sequential Importance Sampling 369
    9.7.6 Response Surface Estimation via Importance Sampling 373
  9.8 Quasi Monte Carlo 376
  References 379
10 Rare-Event Simulation
10.1 Efficiency of Estimators
10.2 Importance Sampling Methods for Light Tails
  10.2.1 Estimation of Stopping Time Probabilities
  10.2.2 Estimation of Overflow Probabilities
  10.2.3 Estimation For Compound Poisson Sums
10.3 Conditioning Methods for Heavy Tails
  10.3.1 Estimation for Compound Sums
  10.3.2 Sum of Nonidentically Distributed Random Variables
10.4 State-Dependent Importance Sampling
10.5 Cross-Entropy Method for Rare-Event Simulation
10.6 Splitting Method
References

11 Estimation of Derivatives
11.1 Gradient Estimation
11.2 Finite Difference Method
11.3 Infinitesimal Perturbation Analysis
11.4 Score Function Method
  11.4.1 Score Function Method With Importance Sampling
11.5 Weak Derivatives
11.6 Sensitivity Analysis for Regenerative Processes
References

12 Randomized Optimization
12.1 Stochastic Approximation
12.2 Stochastic Counterpart Method
12.3 Simulated Annealing
12.4 Evolutionary Algorithms
  12.4.1 Genetic Algorithms
  12.4.2 Differential Evolution
  12.4.3 Estimation of Distribution Algorithms
12.5 Cross-Entropy Method for Optimization
12.6 Other Randomized Optimization Techniques
References

13 Cross-Entropy Method
13.1 Cross-Entropy Method
13.2 Cross-Entropy Method for Estimation
13.3 Cross-Entropy Method for Optimization
  13.3.1 Combinatorial Optimization
CONTENTS

13.3.2 Continuous Optimization
13.3.3 Constrained Optimization
13.3.4 Noisy Optimization
References

14 Particle Methods

14.1 Sequential Monte Carlo
14.2 Particle Splitting
14.3 Splitting for Static Rare-Event Probability Estimation
14.4 Adaptive Splitting Algorithm
14.5 Estimation of Multidimensional Integrals
14.6 Combinatorial Optimization via Splitting
  14.6.1 Knapsack Problem
  14.6.2 Traveling Salesman Problem
  14.6.3 Quadratic Assignment Problem
14.7 Markov Chain Monte Carlo With Splitting
References

15 Applications to Finance

15.1 Standard Model
15.2 Pricing via Monte Carlo Simulation
15.3 Sensitivities
  15.3.1 Pathwise Derivative Estimation
  15.3.2 Score Function Method
References

16 Applications to Network Reliability

16.1 Network Reliability
16.2 Evolution Model for a Static Network
16.3 Conditional Monte Carlo
  16.3.1 Leap–Evolve Algorithm
16.4 Importance Sampling for Network Reliability
  16.4.1 Importance Sampling Using Bounds
  16.4.2 Importance Sampling With Conditional Monte Carlo
16.5 Splitting Method
  16.5.1 Acceleration Using Bounds
References

17 Applications to Differential Equations

17.1 Connections Between Stochastic and Partial Differential Equations
CONTENTS

17.1 Boundary Value Problems 579
17.1.1 Boundary Value Problems 579
17.1.2 Terminal Value Problems 584
17.1.3 Terminal–Boundary Problems 585

17.2 Transport Processes and Equations 587
17.2.1 Application to Transport Equations 589
17.2.2 Boltzmann Equation 593

17.3 Connections to ODEs Through Scaling 597

References 602

Appendix A: Probability and Stochastic Processes 605

A.1 Random Experiments and Probability Spaces 605
A.1.1 Properties of a Probability Measure 607
A.2 Random Variables and Probability Distributions 607
A.2.1 Probability Density 610
A.2.2 Joint Distributions 611
A.3 Expectation and Variance 612
A.3.1 Properties of the Expectation 614
A.3.2 Variance 615
A.4 Conditioning and Independence 616
A.4.1 Conditional Probability 616
A.4.2 Independence 616
A.4.3 Covariance 617
A.4.4 Conditional Density and Expectation 618
A.5 $L^p$ Space 619
A.6 Functions of Random Variables 620
A.6.1 Linear Transformations 620
A.6.2 General Transformations 620
A.7 Generating Function and Integral Transforms 621
A.7.1 Probability Generating Function 621
A.7.2 Moment Generating Function and Laplace Transform 621
A.7.3 Characteristic Function 622
A.8 Limit Theorems 623
A.8.1 Modes of Convergence 623
A.8.2 Converse Results on Modes of Convergence 624
A.8.3 Law of Large Numbers and Central Limit Theorem 625
A.9 Stochastic Processes 626
A.9.1 Gaussian Property 627
A.9.2 Markov Property 628
A.9.3 Martingale Property 629
A.9.4 Regenerative Property 630
A.9.5 Stationarity and Reversibility 631
A.10 Markov Chains 632
| A.10.1 Classification of States | 633 |
| A.10.2 Limiting Behavior | 633 |
| A.10.3 Reversibility | 635 |
| A.11 Markov Jump Processes | 635 |
| A.11.1 Limiting Behavior | 638 |
| A.12 Itô Integral and Itô Processes | 639 |
| A.13 Diffusion Processes | 643 |
| A.13.1 Kolmogorov Equations | 646 |
| A.13.2 Stationary Distribution | 648 |
| A.13.3 Feynman–Kac Formula | 648 |
| A.13.4 Exit Times | 649 |
| References | 650 |

**Appendix B: Elements of Mathematical Statistics** 653

| B.1 Statistical Inference | 653 |
| B.1.1 Classical Models | 654 |
| B.1.2 Sufficient Statistics | 655 |
| B.1.3 Estimation | 656 |
| B.1.4 Hypothesis Testing | 660 |
| B.2 Likelihood | 664 |
| B.2.1 Likelihood Methods for Estimation | 667 |
| B.2.2 Numerical Methods for Likelihood Maximization | 669 |
| B.2.3 Likelihood Methods for Hypothesis Testing | 671 |
| B.3 Bayesian Statistics | 672 |
| B.3.1 Conjugacy | 675 |
| References | 676 |

**Appendix C: Optimization** 677

| C.1 Optimization Theory | 677 |
| C.1.1 Lagrangian Method | 683 |
| C.1.2 Duality | 684 |
| C.2 Techniques for Optimization | 685 |
| C.2.1 Transformation of Constrained Problems | 685 |
| C.2.2 Numerical Methods for Optimization and Root Finding | 687 |
| C.3 Selected Optimization Problems | 694 |
| C.3.1 Satisfiability Problem | 694 |
| C.3.2 Knapsack Problem | 694 |
| C.3.3 Max-Cut Problem | 695 |
| C.3.4 Traveling Salesman Problem | 695 |
| C.3.5 Quadratic Assignment Problem | 695 |
| C.3.6 Clustering Problem | 696 |