

CONTENTS

Preface	xvii	
Acknowledgments	xix	
1	Uniform Random Number Generation	1
1.1	Random Numbers	1
1.1.1	Properties of a Good Random Number Generator	2
1.1.2	Choosing a Good Random Number Generator	3
1.2	Generators Based on Linear Recurrences	4
1.2.1	Linear Congruential Generators	4
1.2.2	Multiple-Recursive Generators	5
1.2.3	Matrix Congruential Generators	6
1.2.4	Modulo 2 Linear Generators	6
1.3	Combined Generators	8
1.4	Other Generators	10
1.5	Tests for Random Number Generators	11
1.5.1	Spectral Test	12
1.5.2	Empirical Tests	14
	References	21

2	Quasirandom Number Generation	25
2.1	Multidimensional Integration	25
2.2	Van der Corput and Digital Sequences	27
2.3	Halton Sequences	29
2.4	Faure Sequences	31
2.5	Sobol' Sequences	33
2.6	Lattice Methods	36
2.7	Randomization and Scrambling	38
	References	40
3	Random Variable Generation	43
3.1	Generic Algorithms Based on Common Transformations	44
3.1.1	Inverse-Transform Method	45
3.1.2	Other Transformation Methods	47
3.1.3	Table Lookup Method	55
3.1.4	Alias Method	56
3.1.5	Acceptance-Rejection Method	59
3.1.6	Ratio of Uniforms Method	66
3.2	Generation Methods for Multivariate Random Variables	67
3.2.1	Copulas	68
3.3	Generation Methods for Various Random Objects	70
3.3.1	Generating Order Statistics	70
3.3.2	Generating Uniform Random Vectors in a Simplex	71
3.3.3	Generating Random Vectors Uniformly Distributed in a Unit Hyperball and Hypersphere	74
3.3.4	Generating Random Vectors Uniformly Distributed in a Hyperellipsoid	75
3.3.5	Uniform Sampling on a Curve	75
3.3.6	Uniform Sampling on a Surface	76
3.3.7	Generating Random Permutations	79
3.3.8	Exact Sampling From a Conditional Bernoulli Distribution	80
	References	83
4	Probability Distributions	85
4.1	Discrete Distributions	85
4.1.1	Bernoulli Distribution	85
4.1.2	Binomial Distribution	86
4.1.3	Geometric Distribution	91
4.1.4	Hypergeometric Distribution	93
4.1.5	Negative Binomial Distribution	94

4.1.6	Phase-Type Distribution (Discrete Case)	96
4.1.7	Poisson Distribution	98
4.1.8	Uniform Distribution (Discrete Case)	101
4.2	Continuous Distributions	102
4.2.1	Beta Distribution	102
4.2.2	Cauchy Distribution	106
4.2.3	Exponential Distribution	108
4.2.4	F Distribution	109
4.2.5	Fréchet Distribution	111
4.2.6	Gamma Distribution	112
4.2.7	Gumbel Distribution	116
4.2.8	Laplace Distribution	118
4.2.9	Logistic Distribution	119
4.2.10	Log-Normal Distribution	120
4.2.11	Normal Distribution	122
4.2.12	Pareto Distribution	125
4.2.13	Phase-Type Distribution (Continuous Case)	126
4.2.14	Stable Distribution	129
4.2.15	Student's t Distribution	131
4.2.16	Uniform Distribution (Continuous Case)	134
4.2.17	Wald Distribution	135
4.2.18	Weibull Distribution	137
4.3	Multivariate Distributions	138
4.3.1	Dirichlet Distribution	139
4.3.2	Multinomial Distribution	141
4.3.3	Multivariate Normal Distribution	143
4.3.4	Multivariate Student's t Distribution	147
4.3.5	Wishart Distribution	148
	References	150
5	Random Process Generation	153
5.1	Gaussian Processes	154
5.1.1	Markovian Gaussian Processes	159
5.1.2	Stationary Gaussian Processes and the FFT	160
5.2	Markov Chains	162
5.3	Markov Jump Processes	166
5.4	Poisson Processes	170
5.4.1	Compound Poisson Process	174
5.5	Wiener Process and Brownian Motion	177
5.6	Stochastic Differential Equations and Diffusion Processes	183
5.6.1	Euler's Method	185
5.6.2	Milstein's Method	187

5.6.3	Implicit Euler	188
5.6.4	Exact Methods	189
5.6.5	Error and Accuracy	191
5.7	Brownian Bridge	193
5.8	Geometric Brownian Motion	196
5.9	Ornstein–Uhlenbeck Process	198
5.10	Reflected Brownian Motion	200
5.11	Fractional Brownian Motion	203
5.12	Random Fields	206
5.13	Lévy Processes	208
5.13.1	Increasing Lévy Processes	211
5.13.2	Generating Lévy Processes	214
5.14	Time Series	219
	References	222
6	Markov Chain Monte Carlo	225
6.1	Metropolis–Hastings Algorithm	226
6.1.1	Independence Sampler	227
6.1.2	Random Walk Sampler	230
6.2	Gibbs Sampler	233
6.3	Specialized Samplers	240
6.3.1	Hit-and-Run Sampler	240
6.3.2	Shake-and-Bake Sampler	251
6.3.3	Metropolis–Gibbs Hybrids	256
6.3.4	Multiple-Try Metropolis–Hastings	257
6.3.5	Auxiliary Variable Methods	259
6.3.6	Reversible Jump Sampler	269
6.4	Implementation Issues	273
6.5	Perfect Sampling	274
	References	276
7	Discrete Event Simulation	281
7.1	Simulation Models	281
7.2	Discrete Event Systems	283
7.3	Event-Oriented Approach	285
7.4	More Examples of Discrete Event Simulation	289
7.4.1	Inventory System	289
7.4.2	Tandem Queue	293
7.4.3	Repairman Problem	296
	References	300

8	Statistical Analysis of Simulation Data	301
8.1	Simulation Data	301
8.1.1	Data Visualization	302
8.1.2	Data Summarization	303
8.2	Estimation of Performance Measures for Independent Data	305
8.2.1	Delta Method	308
8.3	Estimation of Steady-State Performance Measures	309
8.3.1	Covariance Method	309
8.3.2	Batch Means Method	311
8.3.3	Regenerative Method	313
8.4	Empirical Cdf	316
8.5	Kernel Density Estimation	319
8.5.1	Least Squares Cross Validation	321
8.5.2	Plug-in Bandwidth Selection	326
8.6	Resampling and the Bootstrap Method	331
8.7	Goodness of Fit	333
8.7.1	Graphical Procedures	334
8.7.2	Kolmogorov–Smirnov Test	336
8.7.3	Anderson–Darling Test	339
8.7.4	χ^2 Tests	340
	References	343
9	Variance Reduction	347
9.1	Variance Reduction Example	348
9.2	Antithetic Random Variables	349
9.3	Control Variables	351
9.4	Conditional Monte Carlo	354
9.5	Stratified Sampling	356
9.6	Latin Hypercube Sampling	360
9.7	Importance Sampling	362
9.7.1	Minimum-Variance Density	363
9.7.2	Variance Minimization Method	364
9.7.3	Cross-Entropy Method	366
9.7.4	Weighted Importance Sampling	368
9.7.5	Sequential Importance Sampling	369
9.7.6	Response Surface Estimation via Importance Sampling	373
9.8	Quasi Monte Carlo	376
	References	379

10	Rare-Event Simulation	381
10.1	Efficiency of Estimators	382
10.2	Importance Sampling Methods for Light Tails	385
10.2.1	Estimation of Stopping Time Probabilities	386
10.2.2	Estimation of Overflow Probabilities	389
10.2.3	Estimation For Compound Poisson Sums	391
10.3	Conditioning Methods for Heavy Tails	393
10.3.1	Estimation for Compound Sums	394
10.3.2	Sum of Nonidentically Distributed Random Variables	396
10.4	State-Dependent Importance Sampling	398
10.5	Cross-Entropy Method for Rare-Event Simulation	404
10.6	Splitting Method	409
	References	416
11	Estimation of Derivatives	421
11.1	Gradient Estimation	421
11.2	Finite Difference Method	423
11.3	Infinitesimal Perturbation Analysis	426
11.4	Score Function Method	428
11.4.1	Score Function Method With Importance Sampling	430
11.5	Weak Derivatives	433
11.6	Sensitivity Analysis for Regenerative Processes	435
	References	438
12	Randomized Optimization	441
12.1	Stochastic Approximation	441
12.2	Stochastic Counterpart Method	446
12.3	Simulated Annealing	449
12.4	Evolutionary Algorithms	452
12.4.1	Genetic Algorithms	452
12.4.2	Differential Evolution	454
12.4.3	Estimation of Distribution Algorithms	456
12.5	Cross-Entropy Method for Optimization	457
12.6	Other Randomized Optimization Techniques	460
	References	461
13	Cross-Entropy Method	463
13.1	Cross-Entropy Method	463
13.2	Cross-Entropy Method for Estimation	464
13.3	Cross-Entropy Method for Optimization	468
13.3.1	Combinatorial Optimization	469

13.3.2	Continuous Optimization	471
13.3.3	Constrained Optimization	473
13.3.4	Noisy Optimization	476
	References	477
14	Particle Methods	481
14.1	Sequential Monte Carlo	482
14.2	Particle Splitting	485
14.3	Splitting for Static Rare-Event Probability Estimation	486
14.4	Adaptive Splitting Algorithm	493
14.5	Estimation of Multidimensional Integrals	495
14.6	Combinatorial Optimization via Splitting	504
14.6.1	Knapsack Problem	505
14.6.2	Traveling Salesman Problem	506
14.6.3	Quadratic Assignment Problem	508
14.7	Markov Chain Monte Carlo With Splitting	509
	References	517
15	Applications to Finance	521
15.1	Standard Model	521
15.2	Pricing via Monte Carlo Simulation	526
15.3	Sensitivities	538
15.3.1	Pathwise Derivative Estimation	540
15.3.2	Score Function Method	542
	References	546
16	Applications to Network Reliability	549
16.1	Network Reliability	549
16.2	Evolution Model for a Static Network	551
16.3	Conditional Monte Carlo	554
16.3.1	Leap–Evolve Algorithm	560
16.4	Importance Sampling for Network Reliability	562
16.4.1	Importance Sampling Using Bounds	562
16.4.2	Importance Sampling With Conditional Monte Carlo	565
16.5	Splitting Method	567
16.5.1	Acceleration Using Bounds	573
	References	574
17	Applications to Differential Equations	577
17.1	Connections Between Stochastic and Partial Differential Equations	577

17.1.1	Boundary Value Problems	579
17.1.2	Terminal Value Problems	584
17.1.3	Terminal–Boundary Problems	585
17.2	Transport Processes and Equations	587
17.2.1	Application to Transport Equations	589
17.2.2	Boltzmann Equation	593
17.3	Connections to ODEs Through Scaling	597
	References	602

Appendix A: Probability and Stochastic Processes **605**

A.1	Random Experiments and Probability Spaces	605
A.1.1	Properties of a Probability Measure	607
A.2	Random Variables and Probability Distributions	607
A.2.1	Probability Density	610
A.2.2	Joint Distributions	611
A.3	Expectation and Variance	612
A.3.1	Properties of the Expectation	614
A.3.2	Variance	615
A.4	Conditioning and Independence	616
A.4.1	Conditional Probability	616
A.4.2	Independence	616
A.4.3	Covariance	617
A.4.4	Conditional Density and Expectation	618
A.5	L^p Space	619
A.6	Functions of Random Variables	620
A.6.1	Linear Transformations	620
A.6.2	General Transformations	620
A.7	Generating Function and Integral Transforms	621
A.7.1	Probability Generating Function	621
A.7.2	Moment Generating Function and Laplace Transform	621
A.7.3	Characteristic Function	622
A.8	Limit Theorems	623
A.8.1	Modes of Convergence	623
A.8.2	Converse Results on Modes of Convergence	624
A.8.3	Law of Large Numbers and Central Limit Theorem	625
A.9	Stochastic Processes	626
A.9.1	Gaussian Property	627
A.9.2	Markov Property	628
A.9.3	Martingale Property	629
A.9.4	Regenerative Property	630
A.9.5	Stationarity and Reversibility	631
A.10	Markov Chains	632

A.10.1	Classification of States	633
A.10.2	Limiting Behavior	633
A.10.3	Reversibility	635
A.11	Markov Jump Processes	635
A.11.1	Limiting Behavior	638
A.12	Itô Integral and Itô Processes	639
A.13	Diffusion Processes	643
A.13.1	Kolmogorov Equations	646
A.13.2	Stationary Distribution	648
A.13.3	Feynman–Kac Formula	648
A.13.4	Exit Times	649
	References	650
Appendix B: Elements of Mathematical Statistics		653
B.1	Statistical Inference	653
B.1.1	Classical Models	654
B.1.2	Sufficient Statistics	655
B.1.3	Estimation	656
B.1.4	Hypothesis Testing	660
B.2	Likelihood	664
B.2.1	Likelihood Methods for Estimation	667
B.2.2	Numerical Methods for Likelihood Maximization	669
B.2.3	Likelihood Methods for Hypothesis Testing	671
B.3	Bayesian Statistics	672
B.3.1	Conjugacy	675
	References	676
Appendix C: Optimization		677
C.1	Optimization Theory	677
C.1.1	Lagrangian Method	683
C.1.2	Duality	684
C.2	Techniques for Optimization	685
C.2.1	Transformation of Constrained Problems	685
C.2.2	Numerical Methods for Optimization and Root Finding	687
C.3	Selected Optimization Problems	694
C.3.1	Satisfiability Problem	694
C.3.2	Knapsack Problem	694
C.3.3	Max-Cut Problem	695
C.3.4	Traveling Salesman Problem	695
C.3.5	Quadratic Assignment Problem	695
C.3.6	Clustering Problem	696

C.4	Continuous Problems	696
C.4.1	Unconstrained Problems	696
C.4.2	Constrained Problems	697
References		699
Appendix D: Miscellany		701
D.1	Exponential Families	701
D.2	Properties of Distributions	703
D.2.1	Tail Properties	703
D.2.2	Stability Properties	705
D.3	Cholesky Factorization	706
D.4	Discrete Fourier Transform, FFT, and Circulant Matrices	706
D.5	Discrete Cosine Transform	708
D.6	Differentiation	709
D.7	Expectation-Maximization (EM) Algorithm	711
D.8	Poisson Summation Formula	714
D.9	Special Functions	715
D.9.1	Beta Function $B(\alpha, \beta)$	715
D.9.2	Incomplete Beta Function $I_x(\alpha, \beta)$	715
D.9.3	Error Function $\text{erf}(x)$	715
D.9.4	Digamma function $\psi(x)$	716
D.9.5	Gamma Function $\Gamma(\alpha)$	716
D.9.6	Incomplete Gamma Function $P(\alpha, x)$	716
D.9.7	Hypergeometric Function ${}_2F_1(a, b; c; z)$	716
D.9.8	Confluent Hypergeometric Function ${}_1F_1(\alpha; \gamma; x)$	717
D.9.9	Modified Bessel Function of the Second Kind $K_\nu(x)$	717
References		717
Acronyms and Abbreviations		719
List of Symbols		721
List of Distributions		724
Index		727