1. (6 marks) For all integers $a$ and $b$, if $a + b$ is even then $a$ and $b$ are either both even or both odd. Prove this statement
   (i) directly;
   (ii) by contradiction.

2. (4 marks) For all integers $c$, $d$ and $e$, if $c \mid d$ and $c \nmid e$, then $c \nmid (d + e)$.
   Use proof by contradiction to prove this statement.

3. (4 marks) Prove that there exists a unique prime number of the form $n^2 + 2n - 3$, where $n$ is a positive integer.
   Hint: (1) Find some value of $n$ for which the expression is prime. Then you must show uniqueness, so suppose there are two different values of $n$ yielding primes $p$ and $q$; then show that in fact we must have $p = q$.
   (2) Think about primes and factors; what can you do with a quadratic expression like $n^2 + 2n - 3$?

4. (6 marks) Determine whether or not there exist solutions to the following linear Diophantine equations. If a solution exists, give one, that is, give integer values for $x$ and $y$ which satisfy the given equation.
   (a) $14x + 3003y = 7$.
   (b) $18x + 1028y = 3$.
   (c) $221x - 255y = 17$.

This assignment is worth 2%. Marked out of 20; marks allocated as indicated above.