DUE DATE: 5pm Friday 19th September in the designated Assignment box in the Mathematics (Priestley) building (67), for the tutorial group in which you’ve enrolled. Please USE A COVER SHEET!

1. (5 marks) Let $G$, $H$ and $K$ be the graphs below.

(a) Answer the following questions with yes/no, and also give a one line brief explanation for each of your answers.
   (i) $G$ is a simple graph.
   (ii) The total degree of $G$ is 18.
   (iii) $H$ is a subgraph of $G$.
   (iv) $K$ is a subgraph of $G$.
   (v) $H$ is a tree.
   (vi) $G$ contains an Euler circuit.
   (vii) $H$ contains an Euler path.
   (viii) $H$ is bipartite.

(b) Now draw any one subgraph of $G$ which contains all five vertices of $G$ and which is a tree.

2. (4 marks) For each of the following, state whether or not there exists a simple graph with vertices having degrees as stated. If there is no such simple graph, explain why; if a simple graph does exist with the given degrees, then draw such a graph.
   (a) Seven vertices with degrees 6, 5, 4, 3, 2, 2, 1.
   (b) Five vertices with degrees 3, 3, 2, 1, 1.

3. (5 marks)
   (a) If a tree contains exactly 9 vertices, how many edges does it contain? (Explain your answer very briefly.)
   (b) A tree $T$ has 9 vertices. The degrees of its vertices are

   $1, 1, 1, 1, 1, 2, 2, r, s.$

   If $3 \leq r \leq s$, find $r$ and $s$.

4. (6 marks) Which of the following graphs contains an Euler circuit, which contains an Euler path, and which contains neither? (Explain your answers briefly, and give any conditions on $n, q$ and $r$.)

   $K_4, \; K_5, \; K_n, \; K_{2,4}, \; K_{2,q}, \; K_{3,r}.$

This assignment is worth 2%. Marked out of 20; marks allocated as indicated above.