MATH3301  SECOND SEMESTER 2008

Geometry Assignment Two  (Worth 8%)

DUE: By 4pm on Thursday 28th August 2008.

1. (2 marks) In the Taxicab plane, if three distinct points $A$, $B$ and $C$ satisfy

$$d_T(A, C) = d_T(A, B) + d_T(B, C),$$

is it true that $A—B—C$? If so, prove it; if not, give a counter-example.

2. (3 marks) Recall that the supremum distance in the Euclidean plane is given by

$$d_S((x_1, y_1), (x_2, y_2)) = \max\{|x_1 - x_2|, |y_1 - y_2|\}.$$

In the Euclidean plane $(\mathbb{R}^2, \mathcal{L}_E, d_S)$, with the supremum distance, sketch the circle with radius 1 and centre $(0, 0)$.

3. (5 marks) Let $A$ and $B$ be distinct points in a metric geometry. Then $M \in \overline{AB}$ is a midpoint of the line segment $\overline{AB}$ if and only if $AM = MB$. (Remember that here $AM$ means $d(A, M)$.)

(a) If $M$ is a midpoint of $\overline{AB}$, prove that $A—M—B$.
(b) Show that $\overline{AB}$ has a midpoint $M$, and that $M$ is unique.
(c) Let $A = (0, 9)$ and $B = (0, 1)$. Find the midpoint of $\overline{AB}$ where $A$ and $B$ are points of
   (i) the Euclidean plane;
   (ii) the Hyperbolic plane.

4. (2 marks) In the Hyperbolic plane, let $A = (-1, 3)$, $B = (0, 2\sqrt{2})$ and $C = (1, \sqrt{5})$. Is it true that $A—B—C$? Explain your answer.

5. (4 marks)

Let $\mathcal{S}$ denote the set of points of the Missing Strip plane (where the missing points are $\{(x, y) \in \mathbb{R}^2 \mid 0 \leq x < 1\}$).

Find all lines in this plane through the point $(-1, 1)$ which are parallel in the Missing Strip plane to
(i) the line $L_2 \cap \mathcal{S}$;
(ii) the line $L_{-1, 2} \cap \mathcal{S}$.

Marks total 16; your mark will be halved to yield a maximum possible mark of 8% towards the final assessment.

Bonus question (worth a possible extra 1%)\(^1\).

In the Hyperbolic plane, show that $\{(x, y) \in \mathbb{H} \mid x^2 + (y - 5)^2 = 9\}$ is the circle with centre $(0, 4)$ and radius $\ln 2$.

\(^1\)But you can’t get more than 50% for this half of the course!