Binary relation $R$ on a set $A$ is a subset of $A \times A$.

A relation is reflexive if $aRa$, for all $a \in A$.

It is symmetric if $aRa$, then $bRb$, for all $a, b \in A$.

1-1 means both 1-1 onto.

If $f$ is onto $B$, then $f(A) = B$.

If $f(a) = b$, then $a = a_2$.

Consider $f: A \rightarrow B$, $f^{-1}$. Then $a_1 = a_2$.
\[ \{2\} = \{3k+2 \mid k \in \mathbb{Z}\} = \{-4, -1, 2, 5, \ldots\} \]
\[ \{1\} = \{3k+1 \mid k \in \mathbb{Z}\} = \{-2, 1, 4, 7, \ldots\} \]
\[ \{0\} = \{3k \mid k \in \mathbb{Z}\} = \{0, \pm 3, \pm 6, \ldots\} \]

\[ \frac{1}{2} \equiv 7 \pmod{3} \]

\[ \| \overline{A} \| \text{ and } \parallel \overline{B} \parallel \]

\[ \{0\} \text{ is the equivalence class of } 0. \]

\[ P \text{ is the relation where } a \parallel b \text{ if } a + b \text{ is even.} \]
Given: \( E \) is in \( \ang{ABC} \)

Prove: \( A, B, C, D, E \) are collinear.

1. For any \( A, B, E \), \( A, B \) lies on \( \ell_1 \).
2. Every line has at least 2 points.

Construct: \( \ell_1 \) through points \( A, B, E \).

Prove: \( A, B, C, D, E \) are collinear.

Given: \( \ell_1 \); \( E \) is in \( \ang{ABC} \)

Prove: \( A, B, C, D, E \) are collinear.
\[(x - c)^2 + y^2 = r^2\]
$E$ is an incidence geom. $E = (R^2, \mathcal{L})$

So $L = \{a, b, c\}$

(1) Say $P, Q \in \mathcal{L}$, $P \neq Q$. Assume $P, Q$ are on 2 lines.

(2) Say $P \in L_a$ and $Q \in L_b$.

Then $x_1 = x_2 = a$. And $y_1 = y_2 = b$.

So $x_1 = x_2 = a$. And $y_1 = y_2 = b$.

So $L = L_a$.
So \( r = s \) so these are squares.

And \( l_2 = (x_1 - c)^2 + y_1^2 = (x'_2 - c)^2 + y_2^2 \) which implies \( c = \frac{x_1 + x'_2}{2} \). 

Summing, we get \( P \leq \frac{(x_2 - x_1)}{2} \) on one.

So \( y_1 = y_2 \) (since \( h > 0 \)). 

So \( x'_1 = x_2 = a \), so \( y_1 = y_2 = (a - c)^2 + h^2 \). 

(2) Say \( P \notin \mathbb{L} \) and \( \mathbb{L} \) and \( \mathbb{L} \) and \( \mathbb{L} \). 

(3) Say \( P \notin \mathbb{L} \) and \( \mathbb{L} \) and \( \mathbb{L} \) and \( \mathbb{L} \). 

(1) Say \( P \notin \mathbb{L} \) and \( \mathbb{L} \) and \( \mathbb{L} \) and \( \mathbb{L} \). 

So \( (x_1, y_1) = (a, h) \). 

Short: \( P = (a, h) \). 

(1) \( \mathbb{L} = (1, 1) \) or \( \mathbb{L} = (2, 2) \). 

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(5) \( \mathbb{L} = (1, 1) \) or \( \mathbb{L} = (2, 2) \).
\[ l = 5 \]

\[ a = -2, \quad c = -1 \]

\[ V \Rightarrow 2c = -2 \quad \Rightarrow \quad r^2 = c + h = (c-1)^2 + 1 \]

AC is on \( x \)-axis since \( r^2 = c + h = (c-1)^2 + 1 \)

BC is on \( y \)-axis.

\[ AB \parallel x \]

\[ \text{If } A = (0, 2), \quad B = (0, 1), \quad C = (1, 1) \]

There are no common points.
a) The line $c = 3.2$.

b) All lines $c \perp 4$ and pass through $(0,2)$.

c) $c = 3.2$.

If the line passes through $(4,0)$, set $16 - 8c + 0 = 4$.

Thus, parallel to $4$, no point on C or can have $x = 4$.

Graph:

Type II line at $y = 0$.

One variable line is $0$. (Type I)