Q1 Consider the graph $G$ shown above. Does it contain a 1-factor? Does it contain a 1-factorization?

Q2 Use Tutte’s Theorem to determine if the graph $H$ shown above contains a 1-factor. Carefully explain your answer.

Q3 Use the closure of a graph to determine if the graph $K$, shown above, is Hamiltonian.

Q4 Calculate the Turan number $T(p, 4)$, when $p \equiv 1 \mod 3$.

Q5 Prove that for every positive integer $n$,

$$r(3, n) \leq \frac{n^2 + n}{2}.$$  

Q6 Use the list of Ramsey numbers on slides 14 and 19 of the notes to show that

$$\begin{align*}
(i) & \quad r(5, 5) \leq 50, \quad \text{and} \quad (ii) & \quad r(4, 6) \leq 43.
\end{align*}$$

Q7 Let $m \geq 2$ and $n \geq 2$ be integers, and let $p = r(m, n) - 1$. Suppose that each edge of $K_p$ is arbitrarily coloured red or blue. Show that:

(i) $K_p$ contains a red $K_{m-1}$ or a blue $K_n$.

(ii) $K_p$ contains a red $K_m$ or a blue $K_{n-1}$. 