Techniques for translationally invariant matrix product states

Ian McCulloch

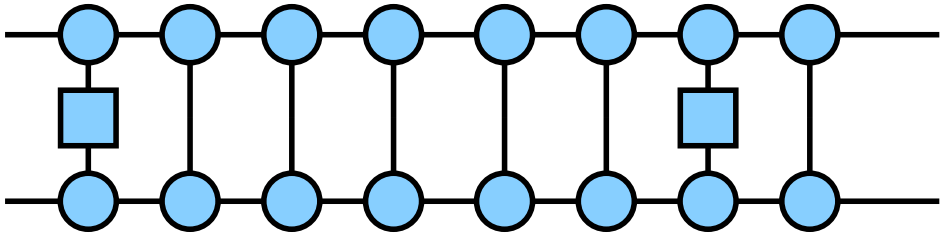
University of Queensland
Centre for Engineered Quantum Systems (EQuS)

12/12/2016

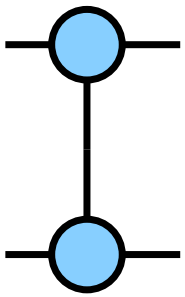
Outline

1. Scaling relations in the thermodynamic limit
2. Expectation values of iMPO's
   - Higher moments
   - Binder cumulant
3. Entanglement spectrum
4. Projective representations
5. Triangular Heisenberg model example
6. Binder ratio for symmetric states

Correlation Functions



The form of correlation functions are determined by the eigenvalues of the *transfer operator*



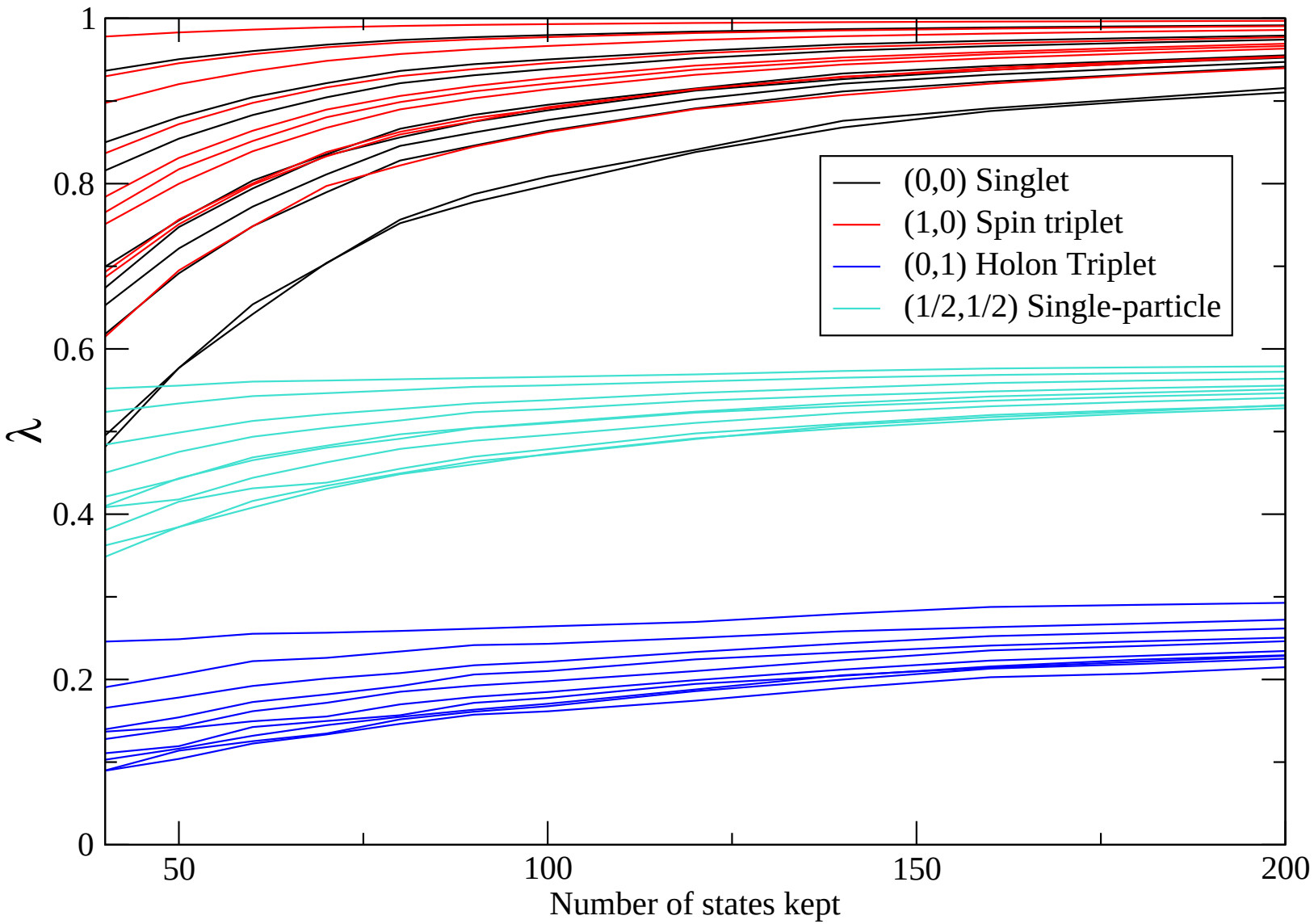
- All eigenvalues ≤ 1
- One eigenvalue equal to 1, corresponding to the identity operator

Expansion in terms of eigenspectrum λ_i:

$$\langle O(x)O(y)\rangle = \sum_i a_i \lambda_i^{|y-x|}$$

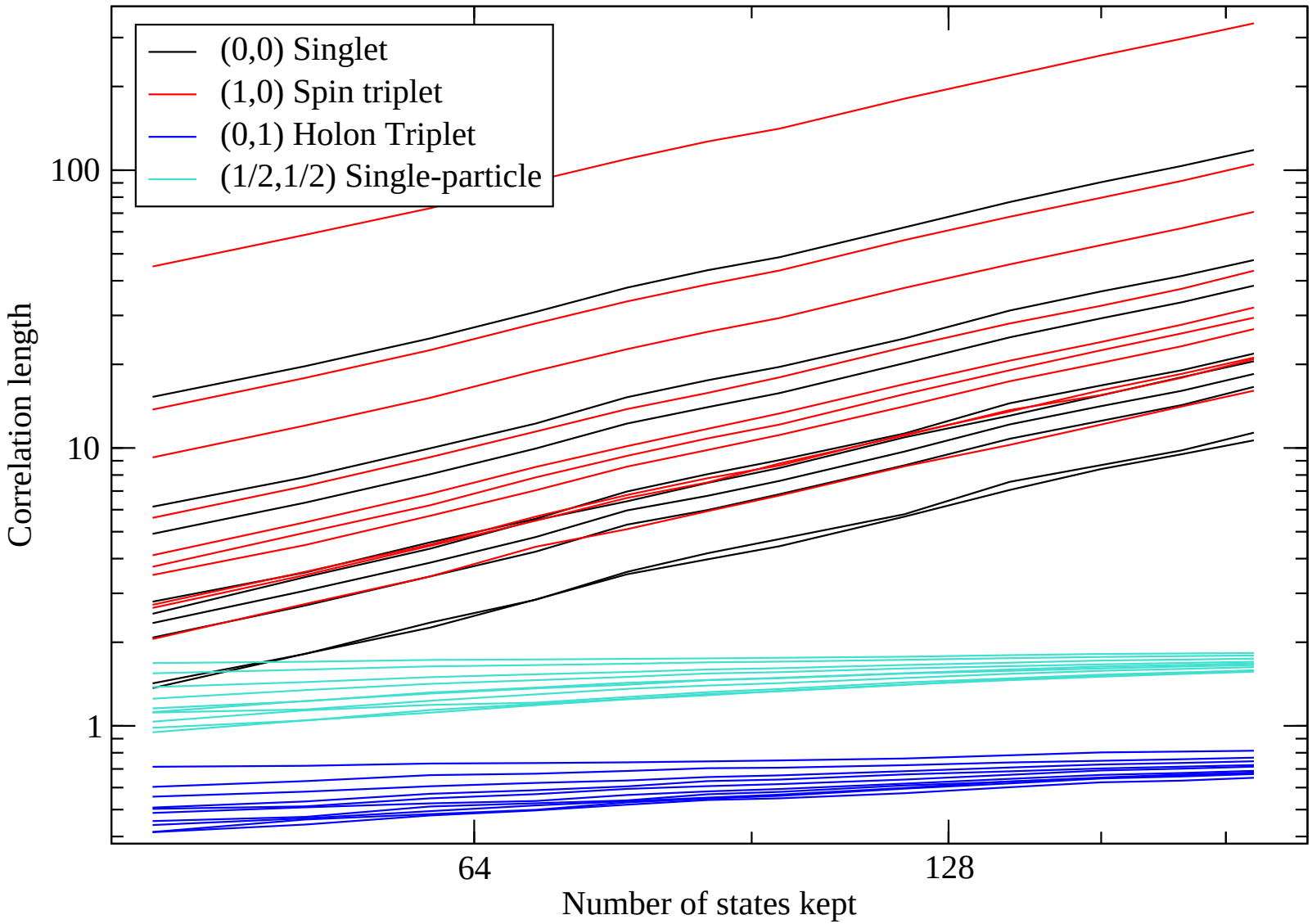
Hubbard Model transfer matrix spectrum

Half-filling, U/t = 4



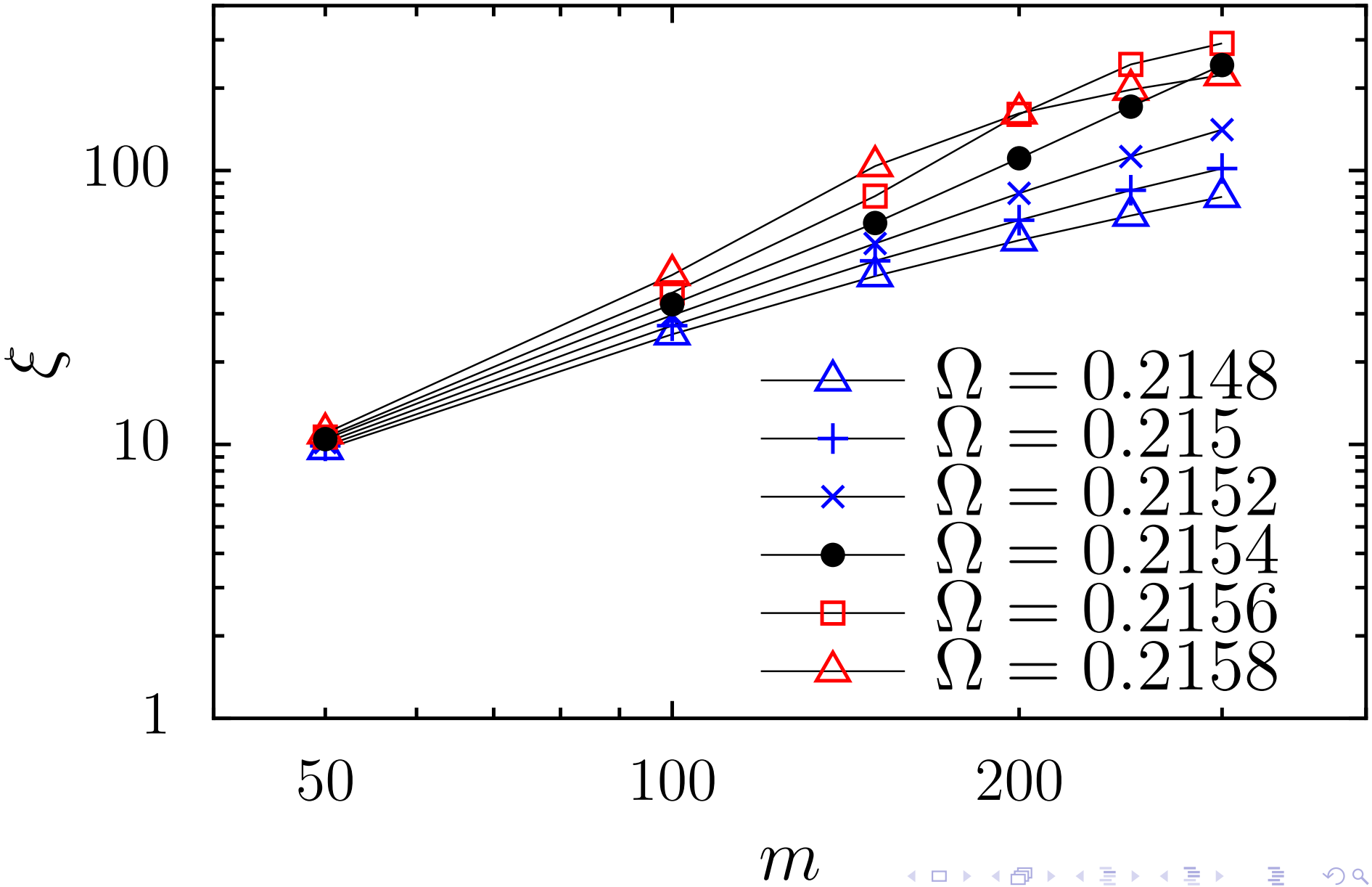
Hubbard model transfer matrix spectrum

Half-filling, U/t=4



Critical scaling example

Two-species bose gas with linear tunneling Ω, from F. Zhan et al, Phys. Rev. A 90, 023630 2014



CFT Parameters

For a critial mode, the correlation length increases with number of states m as a power law,

$$\xi \sim m^{\kappa}$$

[T. Nishino, K. Okunishi, M. Kikuchi, Phys. Lett. A **213**, 69 (1996)
M. Andersson, M. Boman, S. Östlund, Phys. Rev. B **59**, 10493 (1999)
L. Tagliacozzo, Thiago. R. de Oliveira, S. Iblisdir, J. I. Latorre, Phys. Rev. B **78**, 024410 (2008)]

This exponent is a function *only* of the central charge,

$$\kappa = \frac{6}{\sqrt{12c} + c}$$

[Pollmann et al, PRL 2009]

Note: in practice this usually isn't a good way to determine c – better to use entropy scaling

Scaling dimensions

(Vid Stojevic, IPM et al, Phys. Rev. B **91**, 035120 (2015))
Suppose we have a two-point correlator that has a power-law at large distances

$$\langle O(x)O(y)\rangle = |y-x|^{-2\Delta}$$

As we increase the number of states kept m the correlation length increases, so the region of validity of the power law increases.

- Take two different calculations with m_1 and m_2
- Correlation lengths ξ_1 and ξ_2
- We expect: $\dfrac{O(\xi_2)}{O(\xi_1)} = \left(\dfrac{\xi_2}{\xi_1}\right)^{\Delta}$
- for x large, we have: $O(x) \simeq a\,\lambda^x$ (with $\xi = -1/\ln\lambda$)
- Prefactor a is overlap of operator O with next-leading eigenvector of transfer operator

$$a \propto \xi^{-\Delta}$$

This gives directly the operator scaling dimensions by direct fit

Scaling dimensions

(Vid Stojevic, IPM et al, Phys. Rev. B **91**, 035120 (2015))
Suppose we have a two-point correlator that has a power-law at large distances

$$\langle O(x)O(y)\rangle = |y-x|^{-2\Delta}$$

As we increase the number of states kept m the correlation length increases, so the region of validity of the power law increases.

- Take two different calculations with m_1 and m_2
- Correlation lengths ξ_1 and ξ_2
- We expect: $\dfrac{O(\xi_2)}{O(\xi_1)} = \left(\dfrac{\xi_2}{\xi_1}\right)^{\Delta}$
- for x large, we have: $O(x) \simeq a\ \lambda^x$ (with $\xi = -1/\ln\lambda$)
- Prefactor a is overlap of operator O with next-leading eigenvector of transfer operator

$$a \propto \xi^{-\Delta}$$

This gives directly the operator scaling dimensions by direct fit

Scaling dimensions

(Vid Stojevic, IPM et al, Phys. Rev. B **91**, 035120 (2015))
Suppose we have a two-point correlator that has a power-law at large distances

$$\langle O(x)O(y)\rangle = |y-x|^{-2\Delta}$$

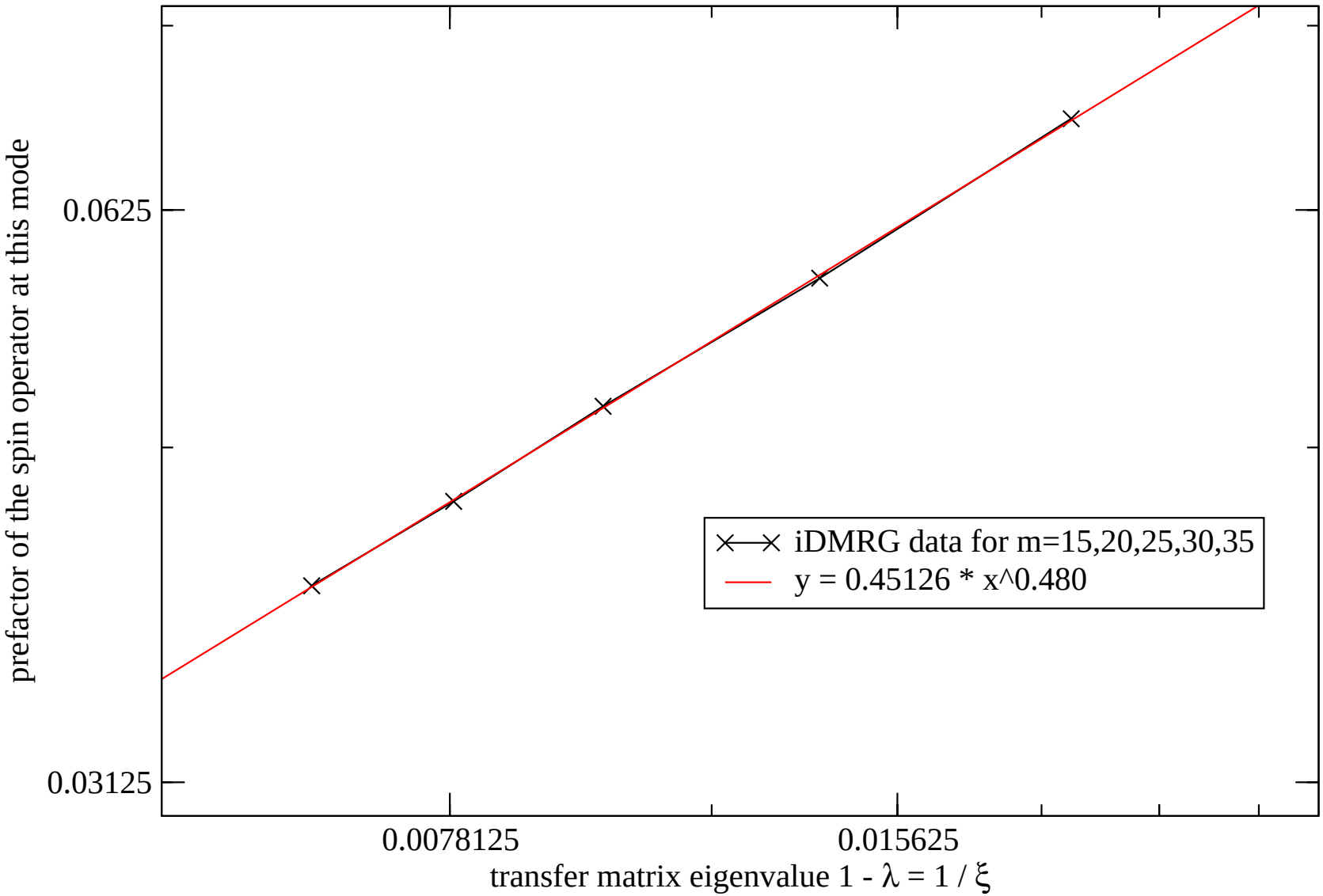
As we increase the number of states kept m the correlation length increases, so the region of validity of the power law increases.

- Take two different calculations with m_1 and m_2
- Correlation lengths ξ_1 and ξ_2
- We expect: $\dfrac{O(\xi_2)}{O(\xi_1)} = \left(\dfrac{\xi_2}{\xi_1}\right)^{\Delta}$
- for x large, we have: $O(x) \simeq a\ \lambda^x$ (with $\xi = -1/\ln\lambda$)
- Prefactor a is overlap of operator O with next-leading eigenvector of transfer operator

$$a \propto \xi^{-\Delta}$$

This gives directly the operator scaling dimensions by direct fit

Heisenberg model fit for the scaling dimension

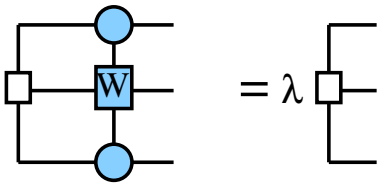


Expectation values of MPO's - arxiv:0804.2509

We have seen that we can write many interesting operators in the form of a matrix product operator

- Can we evaluate the expectation value of an arbitrary MPO?

If the MPO has no Jordan structure, this is a simple eigenvalue problem



For a lower triangular MPO, this doesn't work.

- But we can make use of the triangular structure

$$\begin{pmatrix} I & & \\ S^z & & \\ \lambda S^x & S^z & I \end{pmatrix}$$

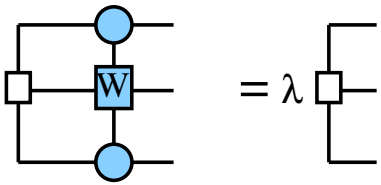
- index by index, each component is a function only of the previously calculated terms

Expectation values of MPO's - arxiv:0804.2509

We have seen that we can write many interesting operators in the form of a matrix product operator

- Can we evaluate the expectation value of an arbitrary MPO?

If the MPO has no Jordan structure, this is a simple eigenvalue problem

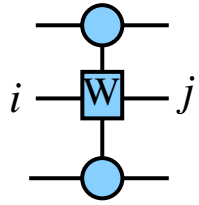


For a lower triangular MPO, this doesn't work.

- But we can make use of the triangular structure

$$\begin{pmatrix} I & & \\ S^z & & \\ \lambda S^x & S^z & I \end{pmatrix}$$

- index by index, each component is a function only of the previously calculated terms



Choose bond indices i, j of W_{ij}, and denote $T_{W_{ij}}$
Example:

$$\begin{pmatrix} I & & \\ S^z & & \\ \lambda S^x & S^z & I \end{pmatrix}$$

Eigentensor is $\begin{pmatrix} E_1 & E_2 & E_3 \end{pmatrix}$

- Starting from E_3:

  $$E_3 = T_I(E_3) = I$$

  is equivalent to the orthogonality condition - E_3 is just the identity
- E_2:

  $$E_2 = T_{S^z}(E_3) = T_{S^z}(I) = S^z$$

- E_1: doesn't reach a fixed point, $E_1 = E_1(L)$ depends on the number of iterations L

  $$\begin{aligned} E_1(L+1) &= T_I(E_1(L)) + T_{S^z}(E_2) + T_{\lambda S^x}(E_3) \\ &= T_I(E_1(L)) + C \end{aligned}$$

  where C is a constant matrix, $C = T_{S^z}(S^z) + \lambda S^x$

Fixed point equations for E_1:

$$E_1(L+1) = T_I(E_1(L)) + C$$

Eigenmatrix expansion of T_I:

$$T_I = \sum_{n=1}^{m^2} \lambda_n |\lambda\rangle\langle\lambda|$$

giving

$$E_1^{(n)}(L+1) = \lambda_n E_1^{(n)}(L) + C^{(n)}$$

- Since $\lambda_1 = 1$ by construction, this motivates decomposing into components parallel (e_1) and perpendicular (E_1') to the identity:

  $$E_1(L+1) = E_1'(L) + e_1(L)\, I$$

  where $\mathrm{Tr}\, E_1'(L)\rho = 0$

- Component in the direction of the identity:

$$e_1(L+1) = e_1(L) + \mathrm{Tr}\, C\rho$$

Has the solution

$$e_1(L) = L\ \mathrm{Tr}\, C\rho$$

is the energy

- Component perpendicular to the identity:

$$E_1'(L+1) = T_I(E_1'(L)) + C'$$

where $C' = C - (\mathrm{Tr}\, C\rho)\, I$

$$E_1'(L+1)_n = \lambda_n E_1'(L)_n + C_n'$$

Since all $|\lambda_n| < 1$ here, this is a geometric series that converges to a fixed point (independent of L),

$$(1 - T_I)(E_1') = C'$$

Linear solver for the unknown matrix E_1'

- Component in the direction of the identity:

$$e_1(L+1) = e_1(L) + \mathrm{Tr}\, C\rho$$

Has the solution

$$e_1(L) = L\ \mathrm{Tr}\, C\rho$$

is the energy

- Component perpendicular to the identity:

$$E_1'(L+1) = T_I(E_1'(L)) + C'$$

where $C' = C - (\mathrm{Tr}\, C\rho)\, I$

$$E_1'(L+1)_n = \lambda_n E_1'(L)_n + C_n'$$

Since all $|\lambda_n| < 1$ here, this is a geometric series that converges to a fixed point (independent of L),

$$(1 - T_I)(E_1') = C'$$

Linear solver for the unknown matrix E_1'

Summary:

- Decompose eigentensor into components parallel and perpendicular to the identity
- The component parallel to the identity is the energy per site
- The perpendicular components reach a fixed point and give the Hamiltonian matrix elements

$$
\begin{array}{lcll}
E_3 & = & I & \text{Identity operator} \\
E_2 & = & Sz & \text{Sz block operator} \\
E_1(L) & = & E' + Le_1 & \text{Hamiltonian operator + energy per site}
\end{array}
$$

Generalization to arbitrary triangular MPO's

arXiv:1008.4667

At the i^{th} iteration, we have

$$E_i(L+1) = T_{W_{ii}}(E_i(L)) + \underbrace{\sum_{j>i} T_{W_{ji}}(E_j(L))}_{= C(L)}$$

Basic idea:

- if $W_{ii} = 0$, then $E_i = C$
- if $W_{ii} \neq 0$, then solve $(1 - T_{W_{ii}})(E_i) = C$

The result will be a polynomial function of L

- solve separately for the coefficient of the k-th power of n
- If the diagonal element is unitary, then obtain the eigenvalues of magnitude 1
- If any eigenvalues are complex, then expand also in fourier modes

Generalization to arbitrary triangular MPO's

arXiv:1008.4667

At the i^{th} iteration, we have

$$E_i(L+1) = T_{W_{ii}}(E_i(L)) + \underbrace{\sum_{j>i} T_{W_{ji}}(E_j(L))}_{= C(L)}$$

Basic idea:

- if $W_{ii} = 0$, then $E_i = C$
- if $W_{ii} \neq 0$, then solve $(1 - T_{W_{ii}})(E_i) = C$

The result will be a polynomial function of L

- solve separately for the coefficient of the k-th power of n
- If the diagonal element is unitary, then obtain the eigenvalues of magnitude 1
- If any eigenvalues are complex, then expand also in fourier modes

Example 1 - Variance

How close is a variational state to an eigenstate of the Hamiltonian?

- Sometimes there is an algorithmic measure, often not.

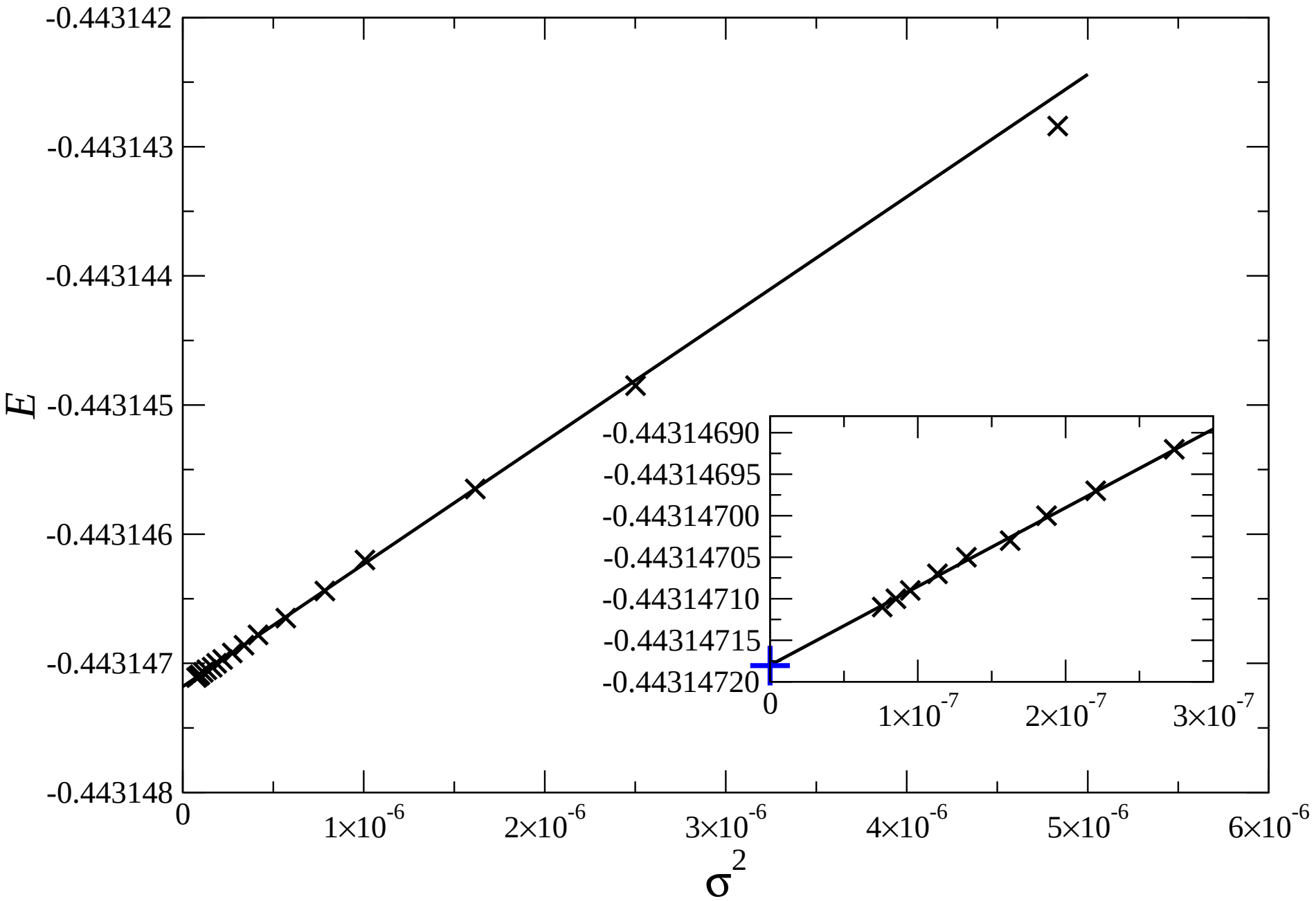
The square of the Hamiltonian operator determines the energy variance

$$\langle H^2 \rangle_L - \langle H \rangle_L^2 = \langle (H - E)^2 \rangle_L = L\sigma^2$$

- A universal measure for the quality of a variational wavefunction
- lower bound for the energy: there is always an eigenstate within σ of E

We can easily construct an MPO representation of H^2

Spin 1/2 Heisenberg Model



Example 2 - momentum distribution

Momentum-dependent operators have a simple form,

$$b_k^\dagger = \sum_x e^{ikx} b_x^\dagger$$

$$W_{b_k^\dagger} = \begin{pmatrix} I & \\ b^\dagger & e^{ik} I \end{pmatrix}$$

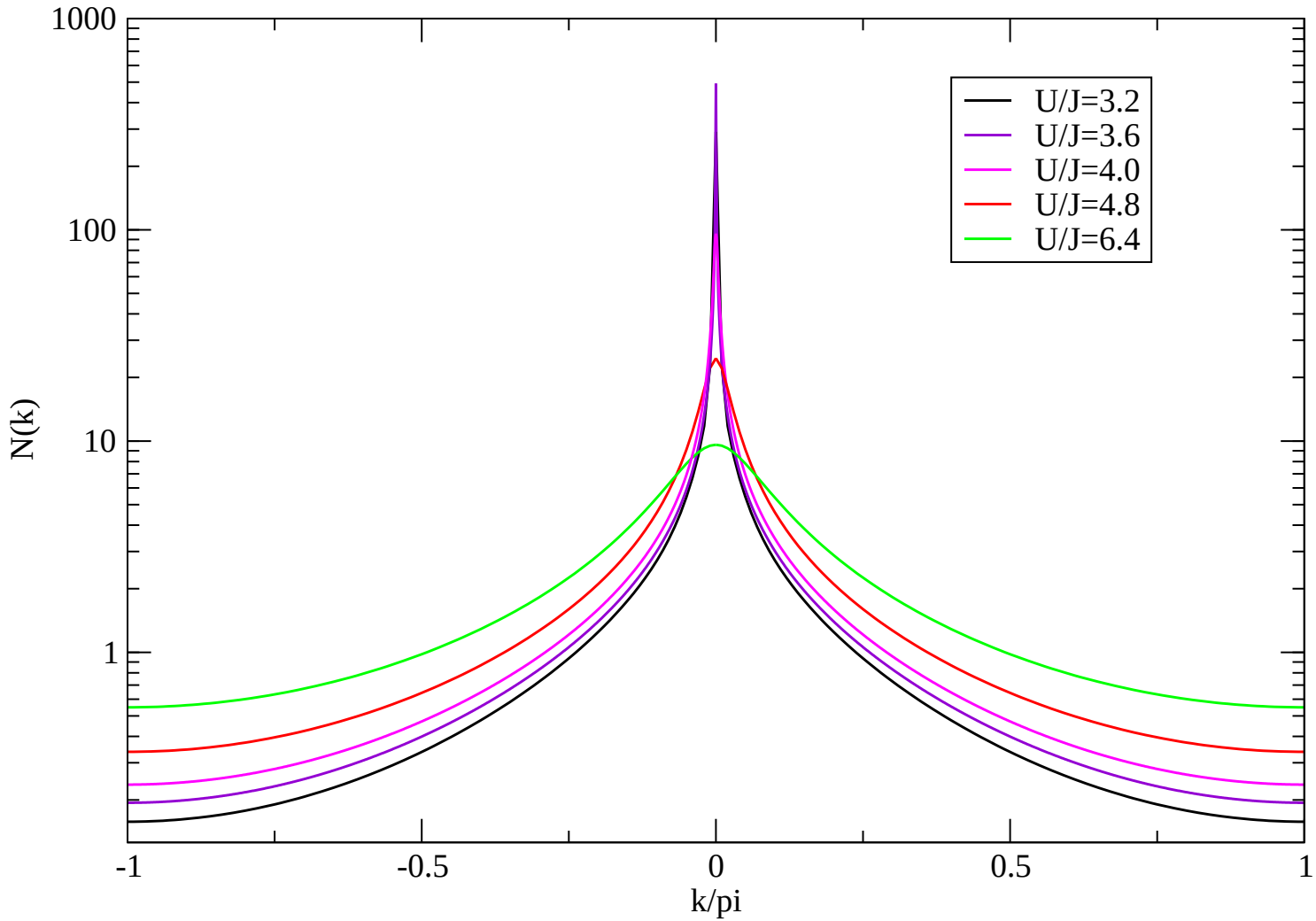
Momentum occupation:

$$n(k) = \frac{1}{L} b_k^\dagger b_k$$

- Broken $U(1)$ symmetry: $\langle b^\dagger \rangle \neq 0$ hence $n(k=0) \propto L$ (extensive)
- With $U(1)$ symmetry: $n(k=0)$ is finite, but diverges with m in superfluid phase

Bose-Hubbard Model N(k)

Infinite 1D, one particle per site



Higher moments

It is straight forward to evaluate a local order parameter, eg

$$M = \sum_i M_i$$

The first moment of this operator gives the order parameter,

$$\langle M \rangle = m_1(L)$$

It is also useful to calculate higher moments, eg

$$\langle M^2 \rangle = m_2(L)$$

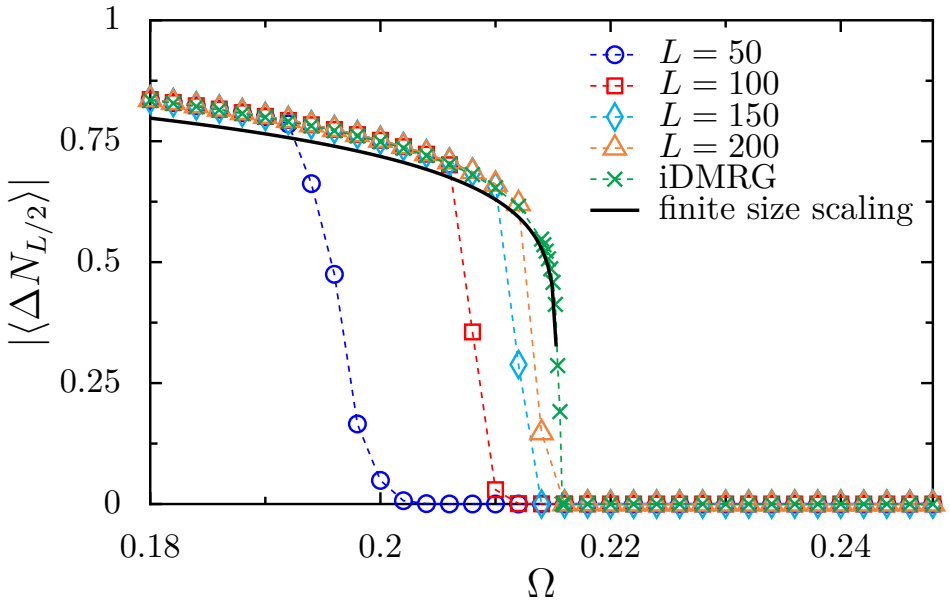
or generally

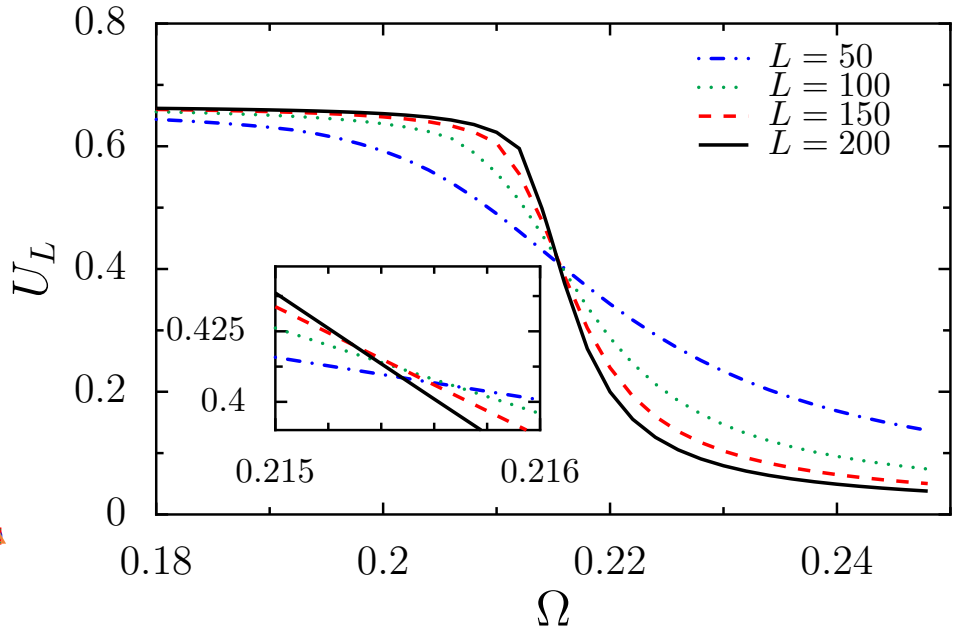
$$\langle M^k \rangle = m_k(L)$$

These are polynomial functions in the system size L.

For *finite* systems, the Binder cumulant of the order parameter cancels the leading-order finite size effects

$$U_L = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}$$





The 2-component Bose-Hubbard model, with a linear coupling between components, has an Ising-like transition from immiscible (small Ω) to miscible (large Ω).

$$H = \sum_{<i,j>,\sigma} b^\dagger_{i,\sigma} b_{j,\sigma} + \text{H.c.} + U \sum_{i,\sigma} n_\sigma (n_\sigma - 1) + U_{12} \sum_i n_\uparrow n_\downarrow + \Omega \sum_{<i,j>} b^\dagger_{i,\uparrow} b_{j,\downarrow} + \text{H.c.}$$

Cumulant expansions

Express the *moments* m_i in terms of the *cumulants per site* κ_j,

$$\begin{aligned} m_1(L) &= \kappa_1 L \\ m_2(L) &= \kappa_1^2 L^2 + \kappa_2 L \\ m_3(L) &= \kappa_1^3 L^3 + 3\kappa_1\kappa_2 L^2 + \kappa_3 L \\ m_4(L) &= \kappa_1^4 L^4 + 6\kappa_1^2\kappa_2 L^3 + (3\kappa_2^2 + 4\kappa_1\kappa_3)L^2 + \kappa_4 L \end{aligned}$$

κ_1 is the order parameter itself
κ_2 is the variance (related to the susceptibility)
κ_3 is the skewness
κ_4 is the kurtosis

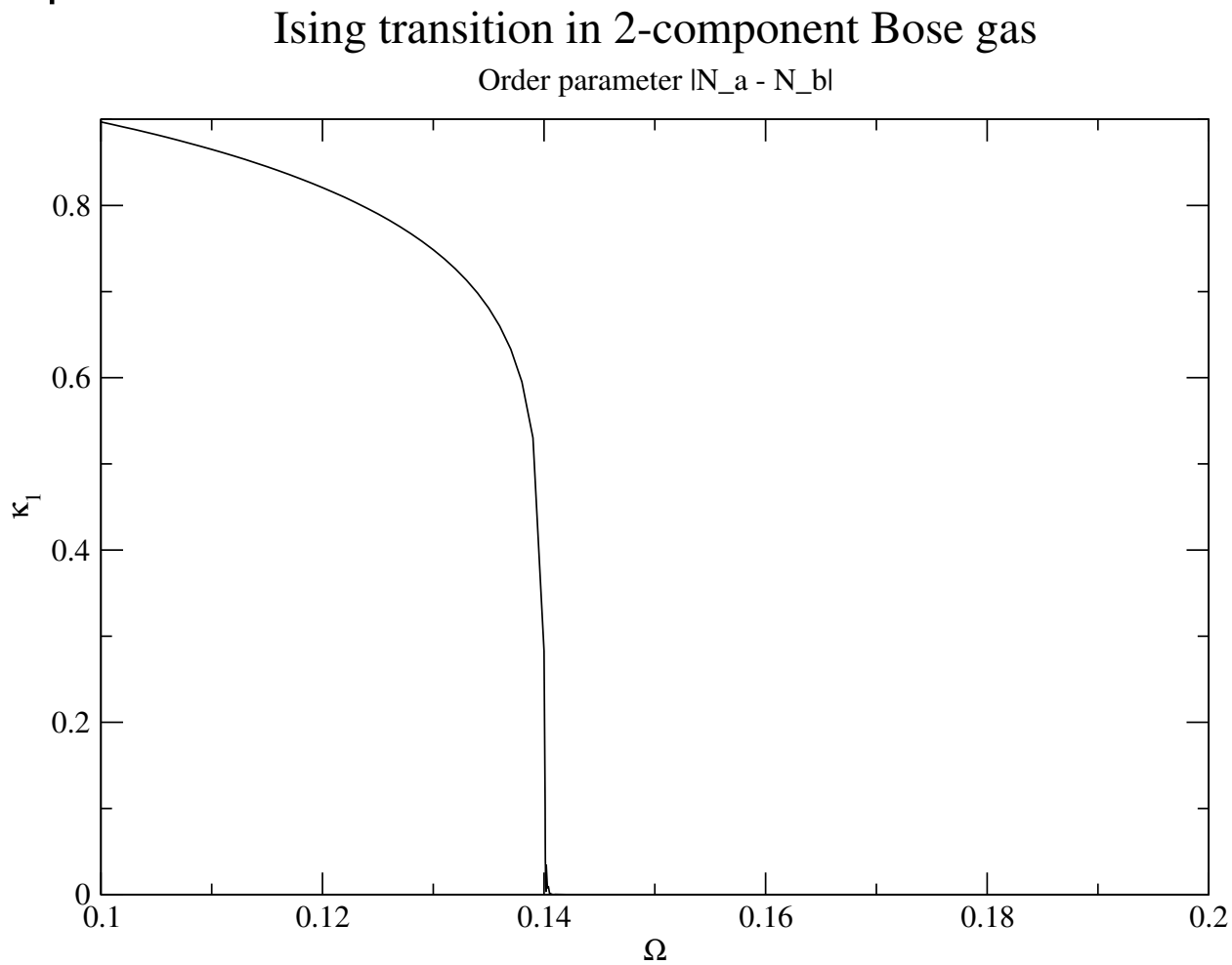
The cumulants per site κ_k are well-defined for an iMPS

Note: the cumulants are normally written such that they are extensive quantities $\rightarrow L\kappa_k$.

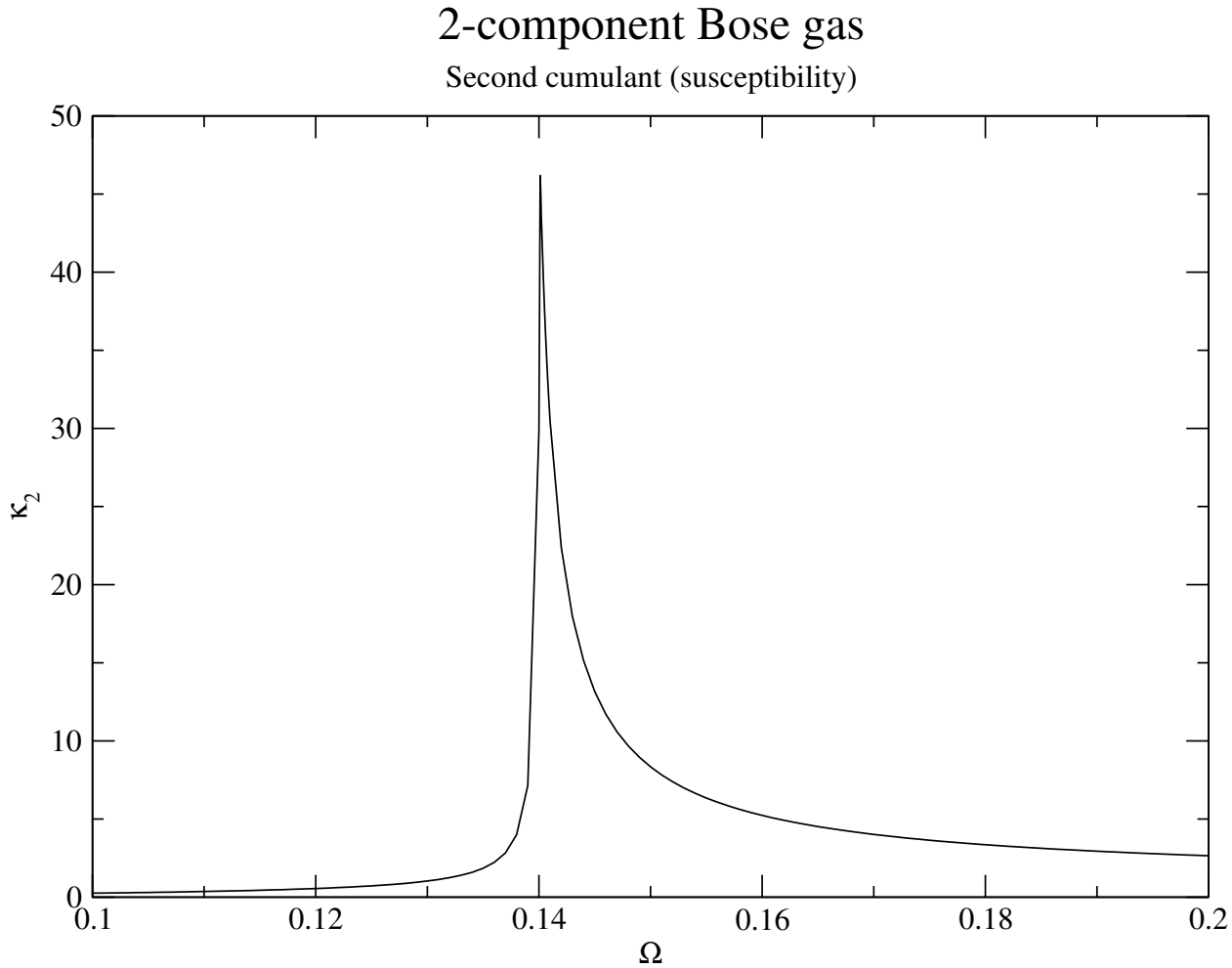
iMPS for two-component bose gas

The cumulant expansion already gives a lot of information

κ_1 is the order parameter itself

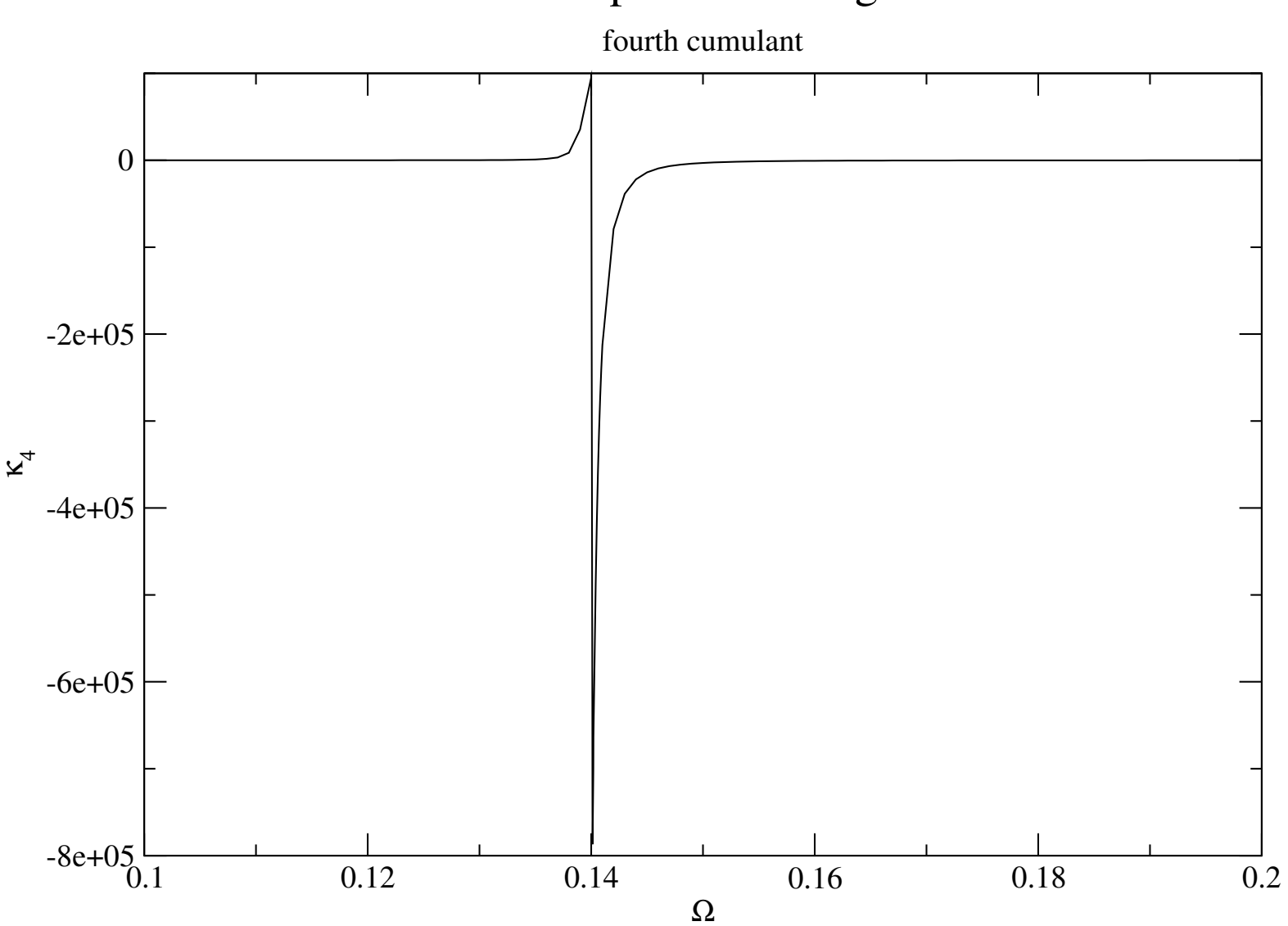


The second cumulant gives the susceptibility



Different to a finite-size scaling, the susceptibility exactly diverges at the critical point.
Sufficiently close to the critical point, it looks mean-field-like (so will generally give the *wrong exponent!*)

The fourth cumulant changes sign at the transition.



Binder Cumulant for iMPS

Naively taking the limit $L \to \infty$ for the Binder cumulant doesn't produce anything useful:

- if the order parameter $\kappa_1 \neq 0$,

$$U_L = 1 - \frac{\langle m^4 \rangle_L}{3\langle m^2 \rangle_L^2} \to \frac{2}{3}$$

- if $\kappa_1 = 0$, then $m_4(L) = 3k_2^2 L^2 + k_4 L$
  Hence

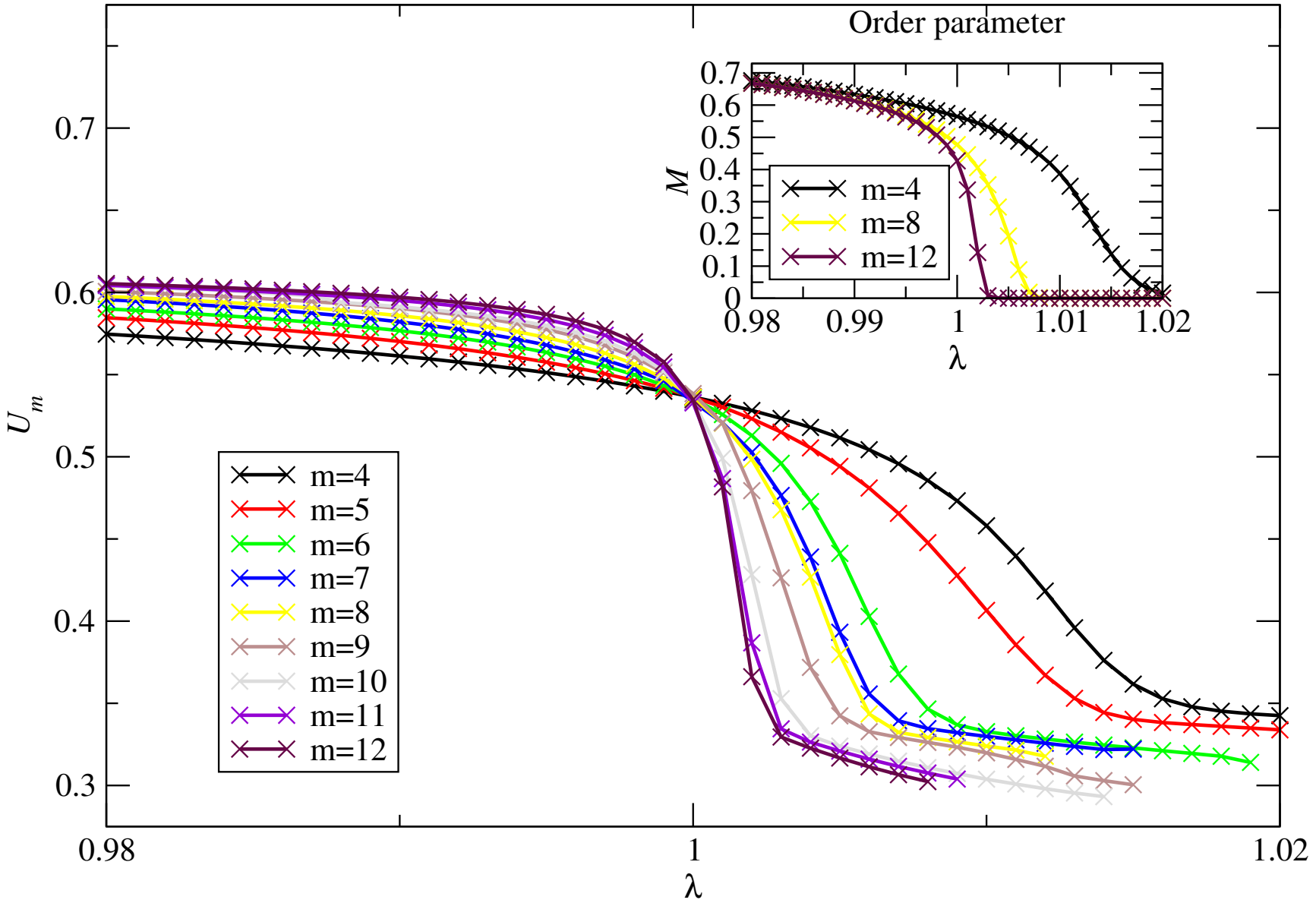
$$U_L = 1 - \frac{3k_2^2 L^2 + k_4 L}{3k_2^2 L^2} \to 0$$

- Finally, a step function that detects whether the order parameter is non-zero

Better approach, in the spirit of finite-entanglement scaling: Evaluate the moment polynomial using $L \propto$ correlation length

Transverse field Ising model

Binder cumulant, scale factor s=5



Scaling functions

Critical exponents

Exponent		relation	
ν	ξ	$\propto$	$\lvert t\rvert^{-\nu}$
β	$\langle M\rangle$	$\propto$	$(-t)^{\beta}$
α	C	$\propto$	$\lvert t\rvert^{\alpha}$
γ	χ	$\propto$	$\lvert t\rvert^{\gamma}$

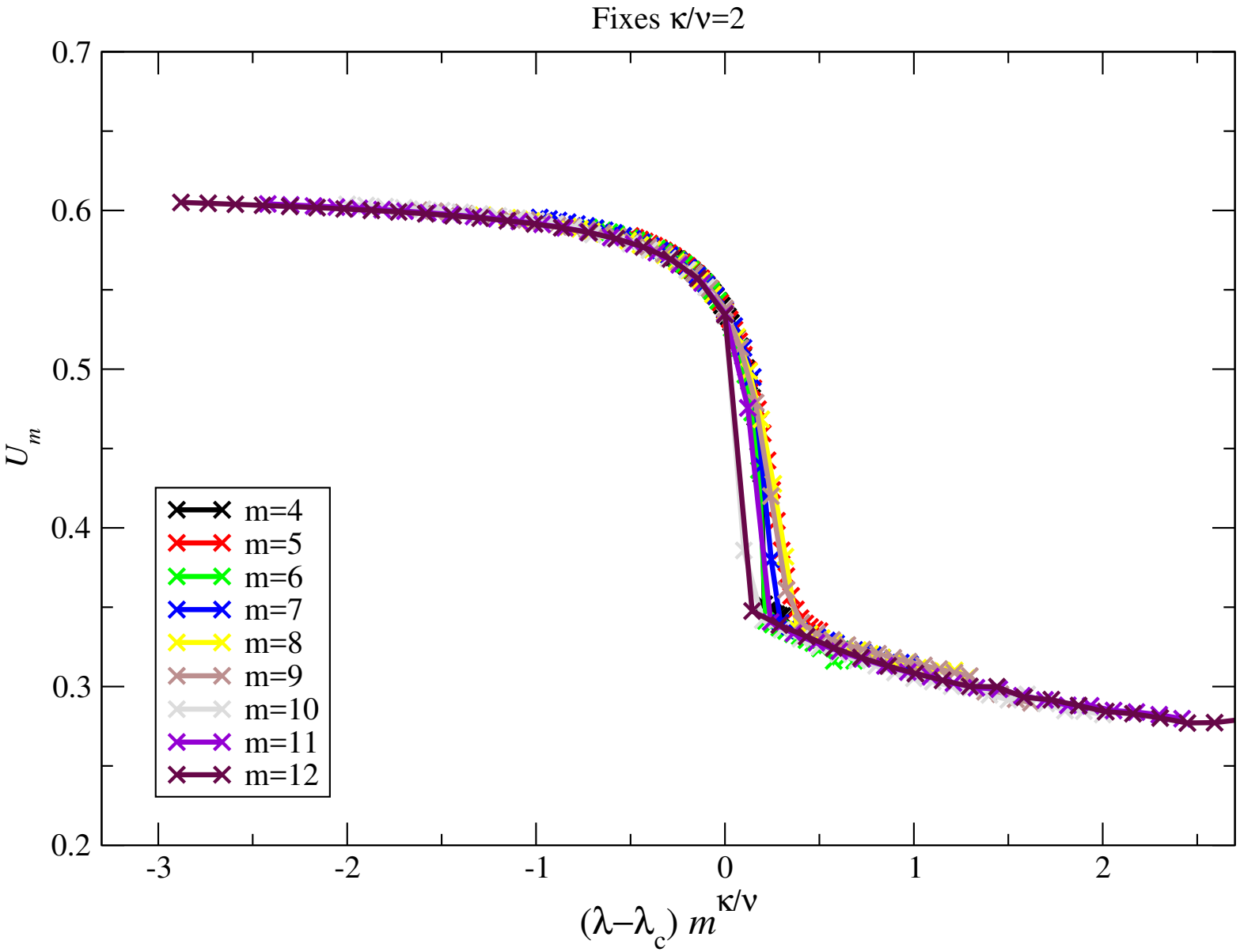
Finite-size scaling functions

$$\begin{aligned} U_L &= \mathcal{U}^0(t\,L^{1/\nu}) \\ \langle M\rangle &= L^{-\beta/\nu}\mathcal{M}^0(t\,L^{1/\nu}) \end{aligned}$$

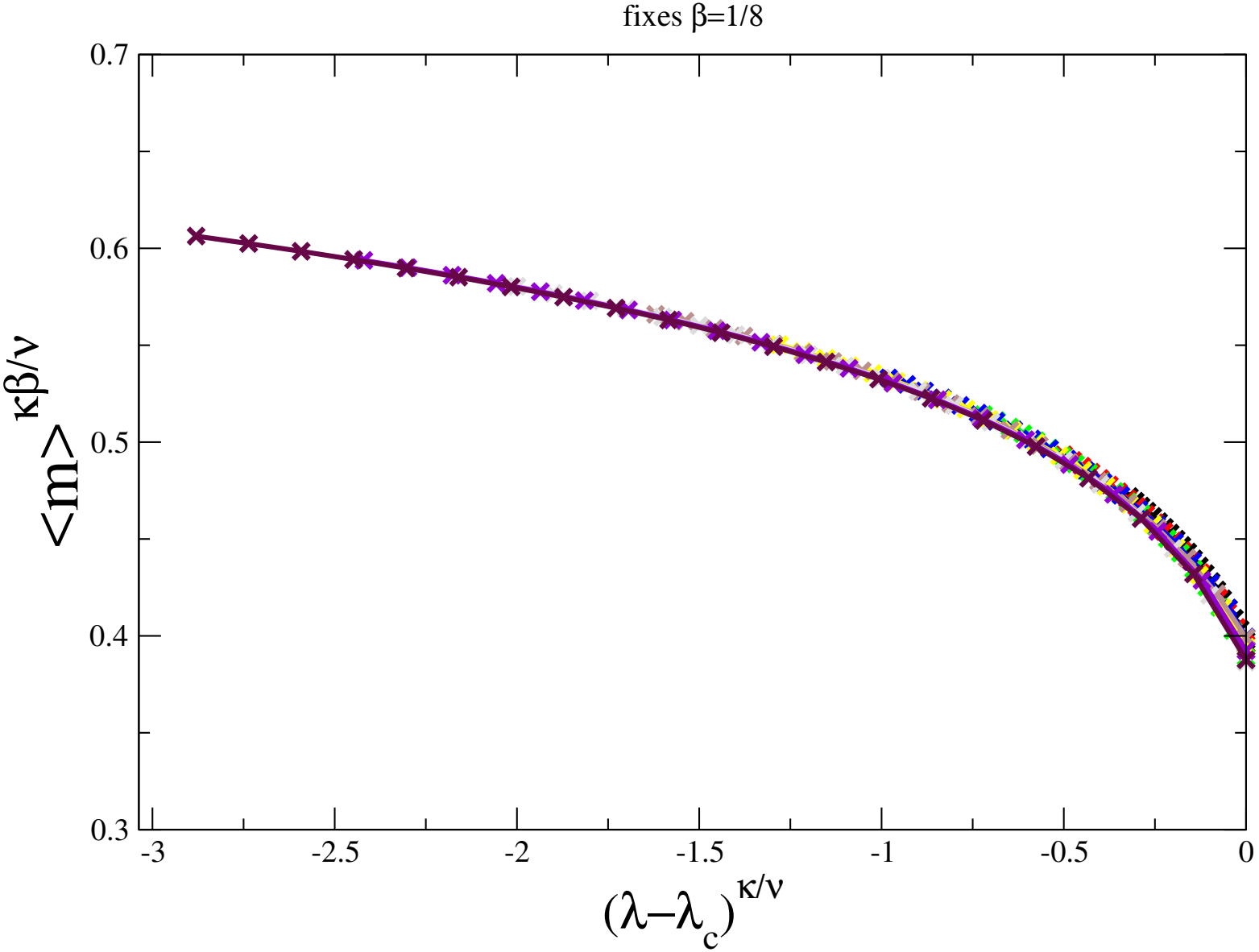
Finite-entanglement scaling functions

$$\begin{aligned} U_m &= \mathcal{U}^0(t\,m^{\kappa/\nu}) \\ \langle M\rangle &= m^{-\beta\kappa/\nu}\mathcal{M}^0(t\,m^{\kappa/\nu}) \end{aligned}$$

Binder Cumulant scaling function collapse



Magnetization scaling function below λ_c



String parameters

Order parameters do not have to be local

Mott insulator string order parameter

$$O_P^2 = \lim_{|j-i|\to\infty} \langle \Pi_{k=i}^{j} (-1)^{n_k} \rangle$$

We can write this as a correlation function of 'kink operators',

$$p_i = \Pi_{k<i} \, (-1)^{n_k}$$

This turns the string order into a 2-point correlation function:

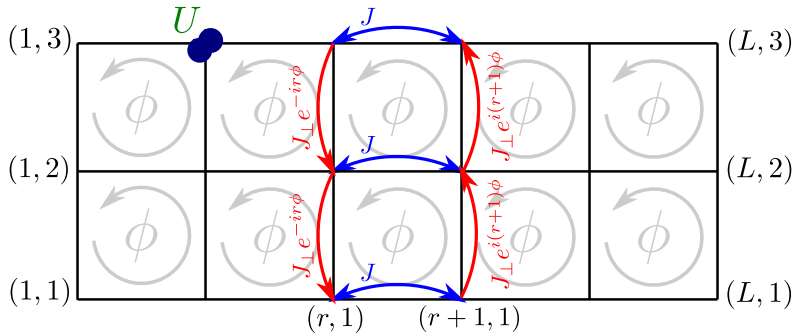
$$O_P^2 = \lim_{|j-i|\to\infty} \langle \, p_i \, p_j \, \rangle$$

Or as an order parameter:

$$P = \sum_i p_i$$

Then $O_p^2 = \frac{1}{L^2} \langle P^2 \rangle$

Real example: 3-leg Bose-Hubbard model with flux phase (F. Kolley, M. Piraud, IPM, U. Schollwoeck, F. Heidrich-Meisner, New J. Phys. 17 (2015) 092001)



For density $n = 1/3$ (one particle per rung), near flux $\phi \sim \pi$, there is a transition from a Mott to critical as a function of $J_\perp$

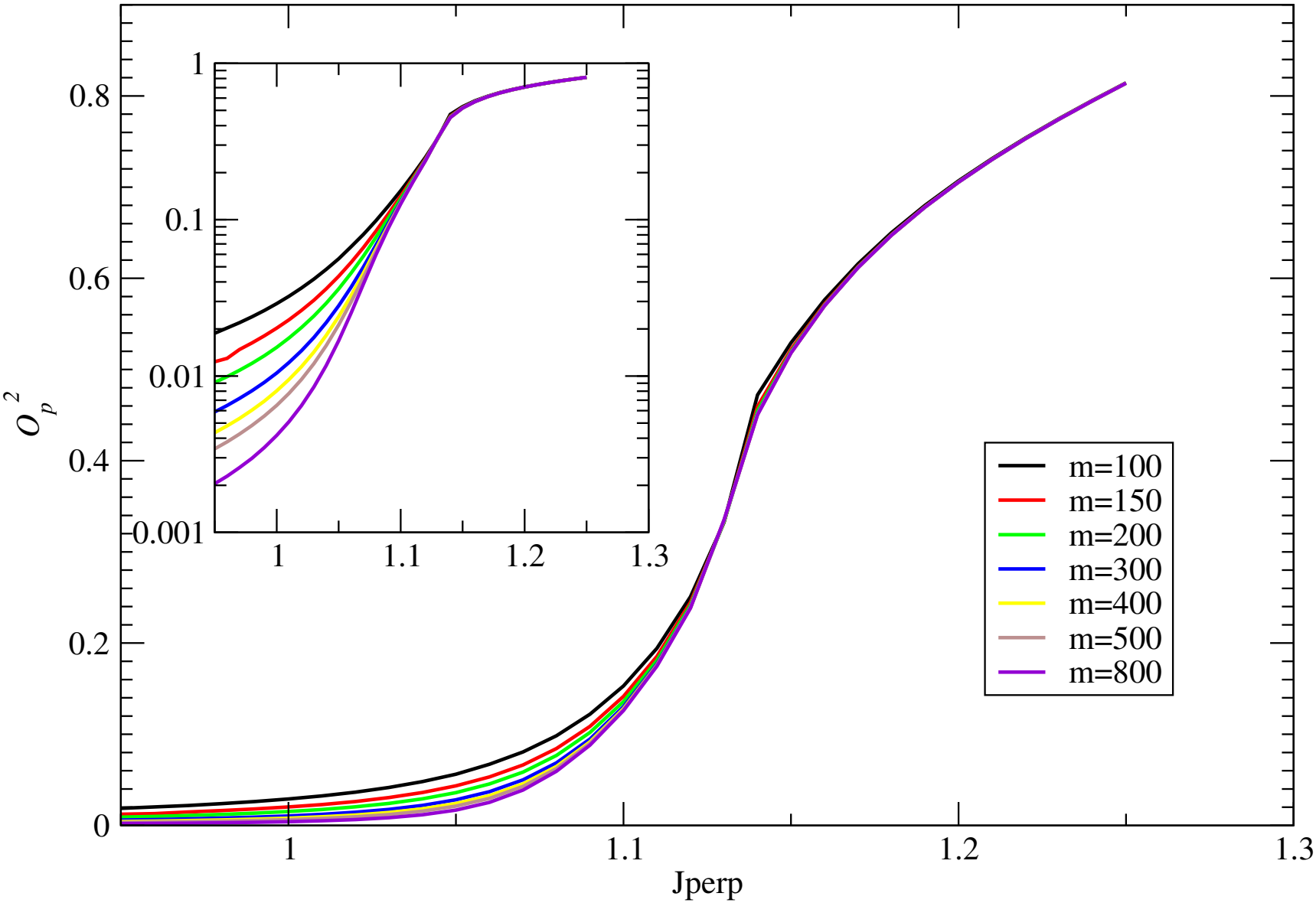
- P has a simple MPO representation

$$P = \begin{pmatrix} I & \\ I & (-1)^n \end{pmatrix}$$

- Hence we can calculate higher moments of P.

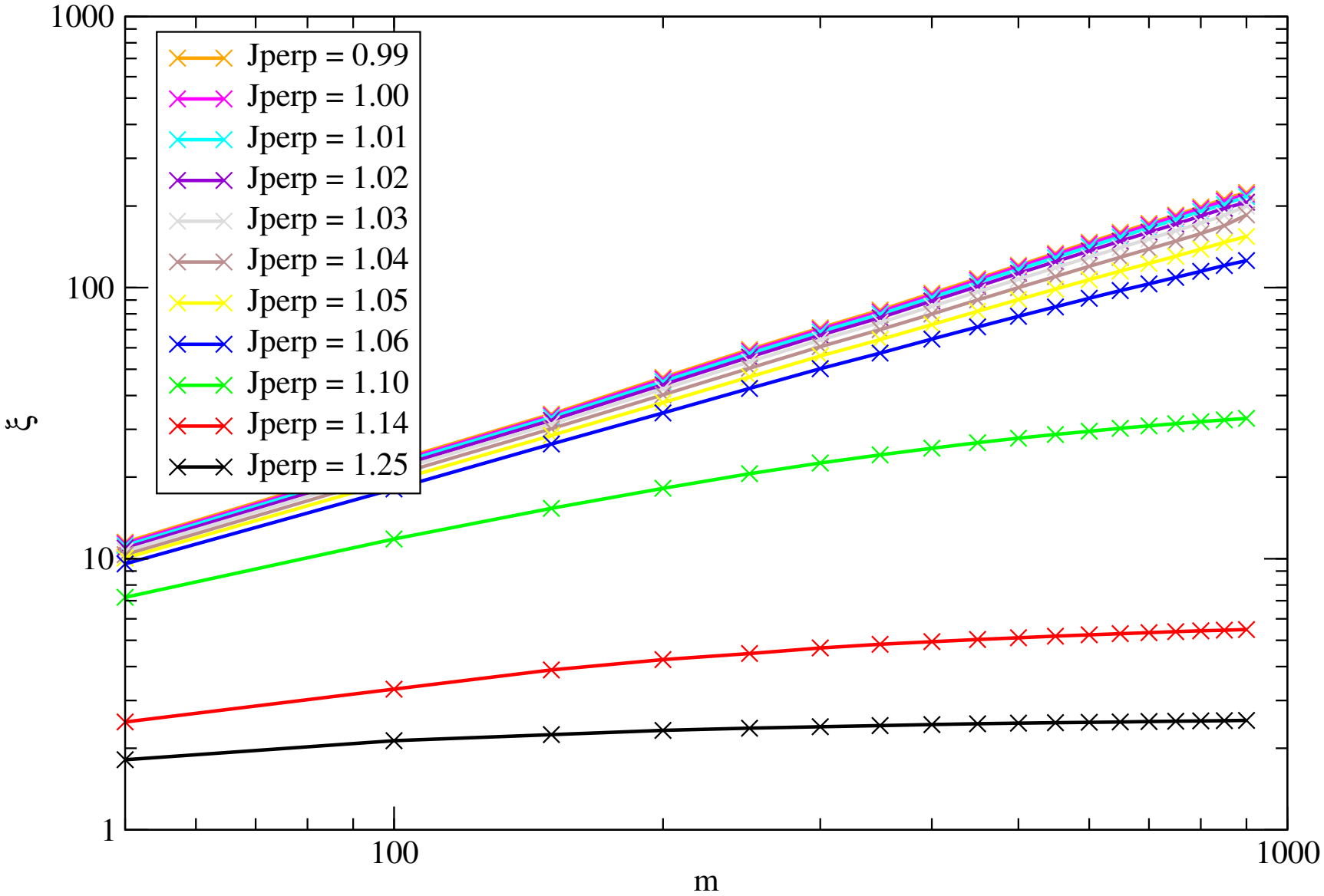
Bose-Hubbard Ladder

String order parameter



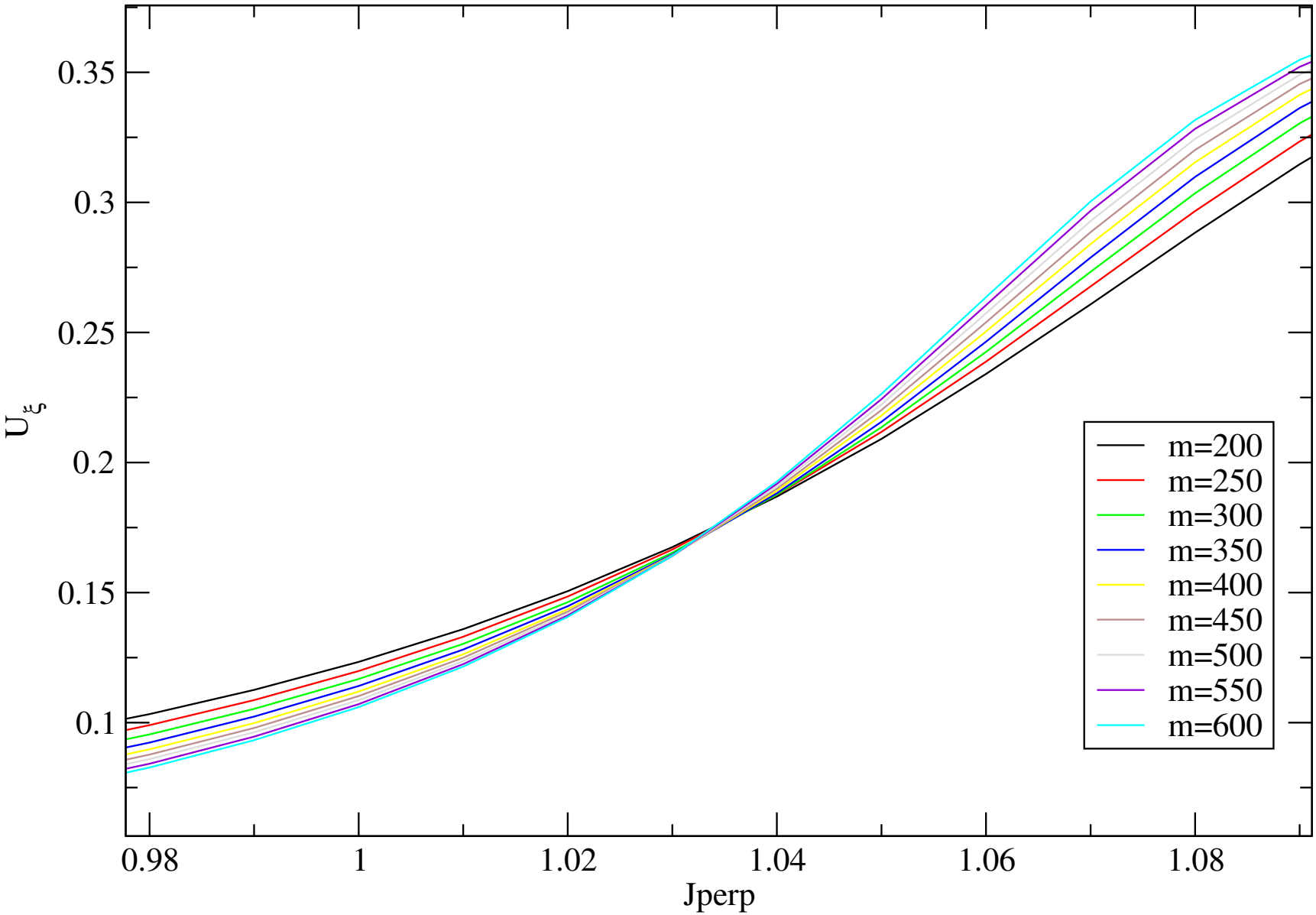
Bose=Hubbard Ladder

Scaling of correlation length

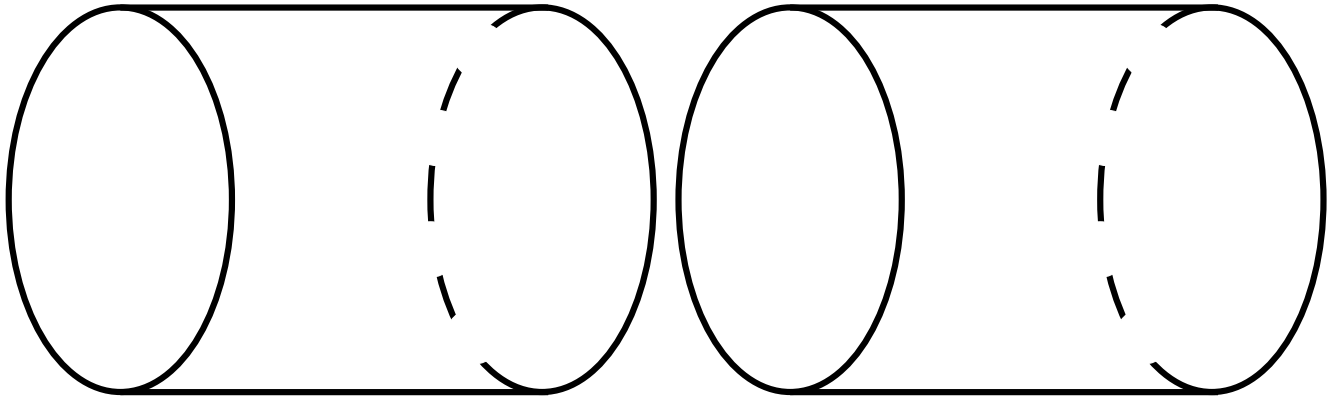


Bose-Hubbard Ladder

String parameter Binder cumulant



Entanglement spectrum spectroscopy



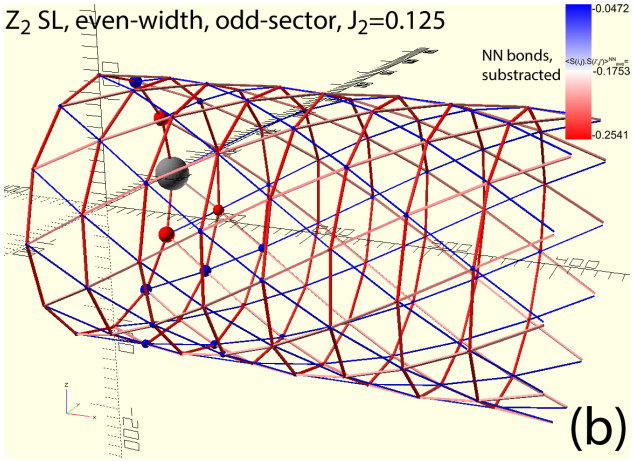
- Bipartite cut – reduced density matrix ρ
- Entanglement Hamiltonian $H = -\ln \rho$
- Reduced dimension – the entanglement Hamiltonian lives on the boundary
- Label states by global symmetries spin, transverse momentum
- Degeneracy structure provides information about symmetries

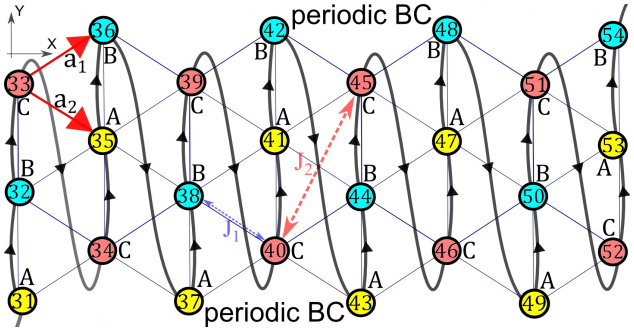
For $SU(2)$ symmetry broken states:

- If the symmetry is allowed to break: bulk gap
- unbroken $SU(2)$: Goldstone mode, separated band of excitations below the bulk gap

Symmetries

Infinite cylinder of circumference L_y





Map onto one-dimensional chain:

Global symmetries

Preserved in the MPS structure:

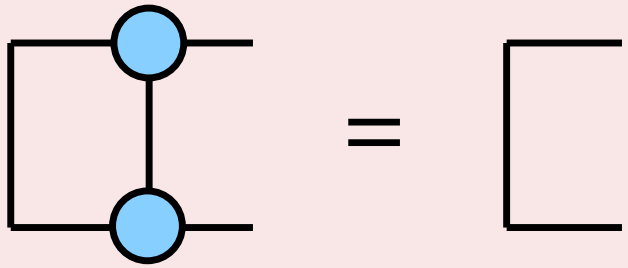
- internal symmetries, such as particle number, spin, . . .

Broken by the real-space structure:

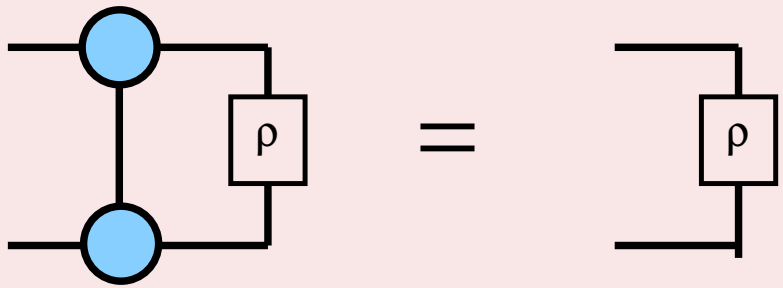
- spatial symmetries, such as translation, reflection of the cylinder

density matrix as the eigenvector of the transfer operator

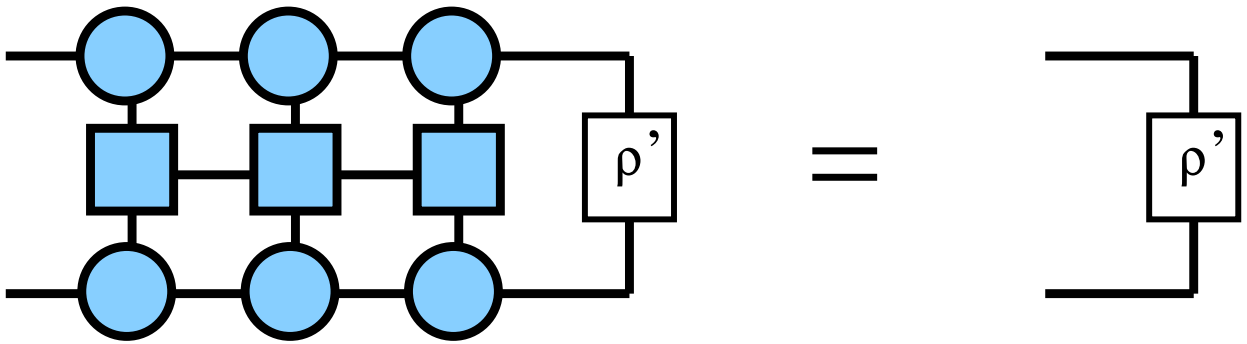
Left eigenvector
identity matrix



Right eigenvector
density matrix



Symmetry-resolved density matrix



Eigenvalues of ρ' are $\lambda_i e^{i\theta_i}$

- λ_i are the density matrix eigenvalues
- θ_i is the corresponding symmetry angle up to a global phase

Allowed symmetries of the ES

Let U be a global symmetry of the wavefunction

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

What is the action of U on the reduced density matrix ρ?

Bipartition:

$$\begin{aligned} |\psi\rangle &= \sum_{ab} \psi_{ab} \quad |a\rangle \otimes |b\rangle \\ U &= U_A \otimes U_B \end{aligned}$$

Action on ρ is $\quad U_A \rho U_A^\dagger$

- U_A is distinguishable only up to a global phase
- U_A is a projective representation of the symmetry

Projective representations

Physical states are *rays* in Hilbert space

- $a|\psi\rangle$, for $a \in \mathbb{C}$ represents the same physical state

Symmetry transformations that are ambiguous up to some scalar are *projective representations*
Assuming a unitary representation, only the phase is unknown: $a = e^{i\theta}$.
A projective representation is defined by:

$$U_x U_y = e^{i\theta(x,y)} U_{xy}$$

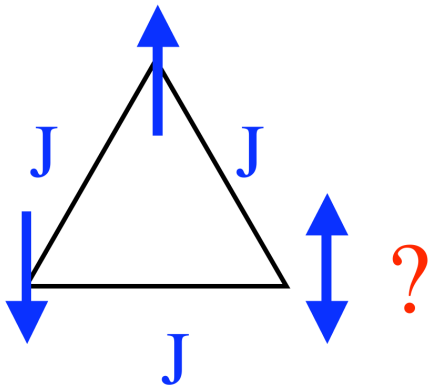
The function $\theta(x, y)$ defines the representation
Example:

- Z_2 has no non-trivial (unitary) projective representations
- $Z_2 \times Z_2$ has has two projective representations

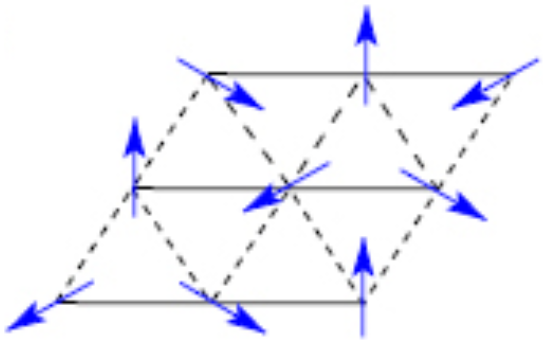
$$\begin{aligned} U_x^2 &= I \\ U_y^2 &= I \\ U_x U_y &= \pm U_y U_x \end{aligned}$$

Triangular lattice – frustration

- Frustrated lattice – suppress ordering



- But ordering may be possible
- Groundstate widely established as $SU(2)$-broken 120° order



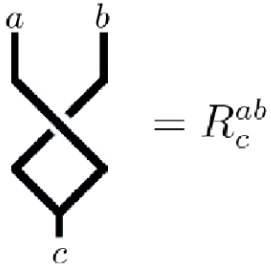
Beyond nearest neighbor: $H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \vec{S}_j$

Z_2 spin liquid

- The simplest spin liquid that doesn't break time reversal symmetry
- Elementary excitations are spinons with non-trivial braiding statistics (anyons)
- Four anyon types:
  - 1 : vacuum
  - b : bosonic spinon (electric charge e)
  - v : vison (magnetic charge m)
  - f : fermionic spinon (fermion ψ)
- Fusion rules for combining excitations:
  - $b \times b = 1$
  - $v \times v = 1$
  - $f \times f = 1$
  - $b \times v = f$
  - $b \times f = v$
  - $v \times f = b$

Anyonic statistics

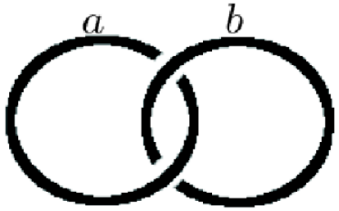
R matrix: counter-clockwise exchange of particles



$$= R_c^{ab}$$

S matrix: intertwining two vacuum excitations

$$S^{ab} = \frac{1}{D}$$



$$S^{ab} = \frac{1}{D} \sum_c d_c \, \mathrm{Tr}\left(R_c^{ab} R_c^{ba}\right)$$

$$R_1^{bb} = R_1^{vv} = 1$$

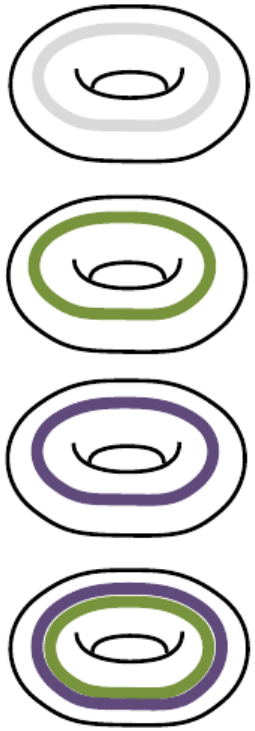
$$R_1^{ff} = (R_f^{vb})^2 = -1$$

In basis $(1, b, v, f)$:

$$S = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Identifying the topology

Groundstate degeneracy on a torus



- construct a vison or spinon that threads down the torus
- spinon flips between time-reversal symmetric and antisymmetric states
- vison inserts -1 flux – antiperiodic boundary conditions
- Symmetry fractionalization – these correspond to *projective* symmetries

How to construct these fluxes?

- translationally invariant infinite system and manipulate the boundary conditions

Antiperiodic boundary conditions

Dihedral group symmetries around the cut:

- T_y – translation by one site around the cylinder
- R_y – reflection about a site or bond

This forms the dihedral group $D_{2\times L_y}$

$$T_y^{L_y} = R_y^2 = 1$$

$$R_y T_y^{L_y/2} R_y^{-1} = T_y^{L_y/2}$$

(a π rotation commutes with a reflection)

The dihedral group has a non-trivial projective representation:

$$T_y^{L_y} = R_y^2 = 1$$

$$R_y T_y R_y^{-1} = -T_y^{-1}$$

(a π rotation anti-commutes with a reflection)
This is the subgroup of $D_{2\times 2\times L_y}$ taking only the odd momenta (antiperiodic BC)

Antiperiodic boundary conditions

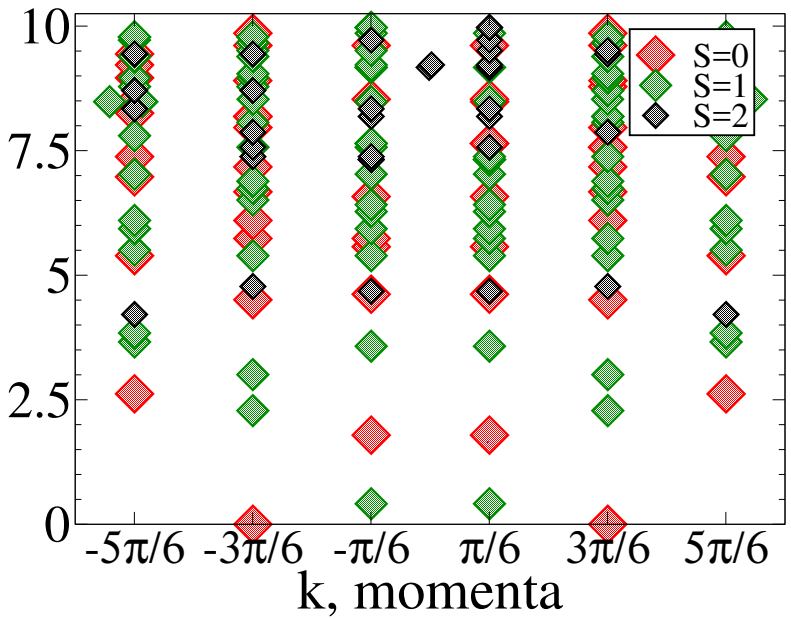
Nariman Saadatmand, IPM, Phys. Rev. B 94, 121111 (2016)

We see two (nearly) degenerate states

- smaller L_y: the groundstate has a 2-fold degenerate entanglement spectrum
- for larger L_y, we obtained a non-degenerate entanglement spectrum

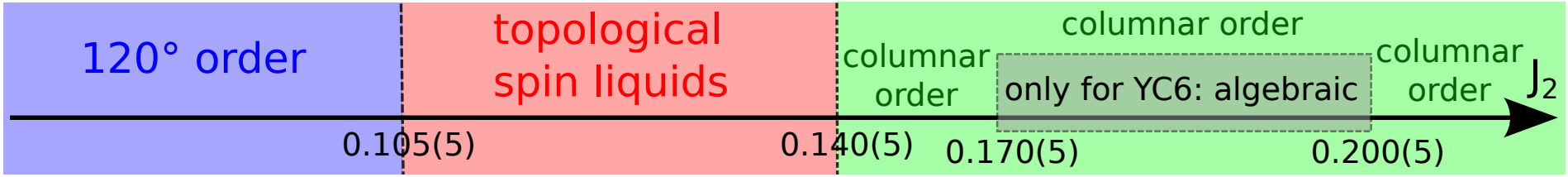
In the two-fold degenerate sector, we measure

$$\langle R_y T_y^{L_y/2} R_y^\dagger T_y^{L/2\dagger} \rangle = -0.9999970\cdots$$



$L_y = 6$ entanglement spectrum

Detecting ordered phases



Using $SU(2)$ symmetry the magnetic order parameter is identically zero.

- can we see signatures of symmetry breaking in higher moments of the order parameter?
- Binder cumulant $U_4 = 1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle}$

Cumulant expansion of the 4th moment:

$$\langle M^4 \rangle_L = \kappa_1^4 L^4 + 6\kappa_2\kappa_1^2 L^3 + (4\kappa_1\kappa_3 + 3\kappa_2^2)L^2 + \kappa_4 L$$

Symmetry implies all *odd* cumulants vanish

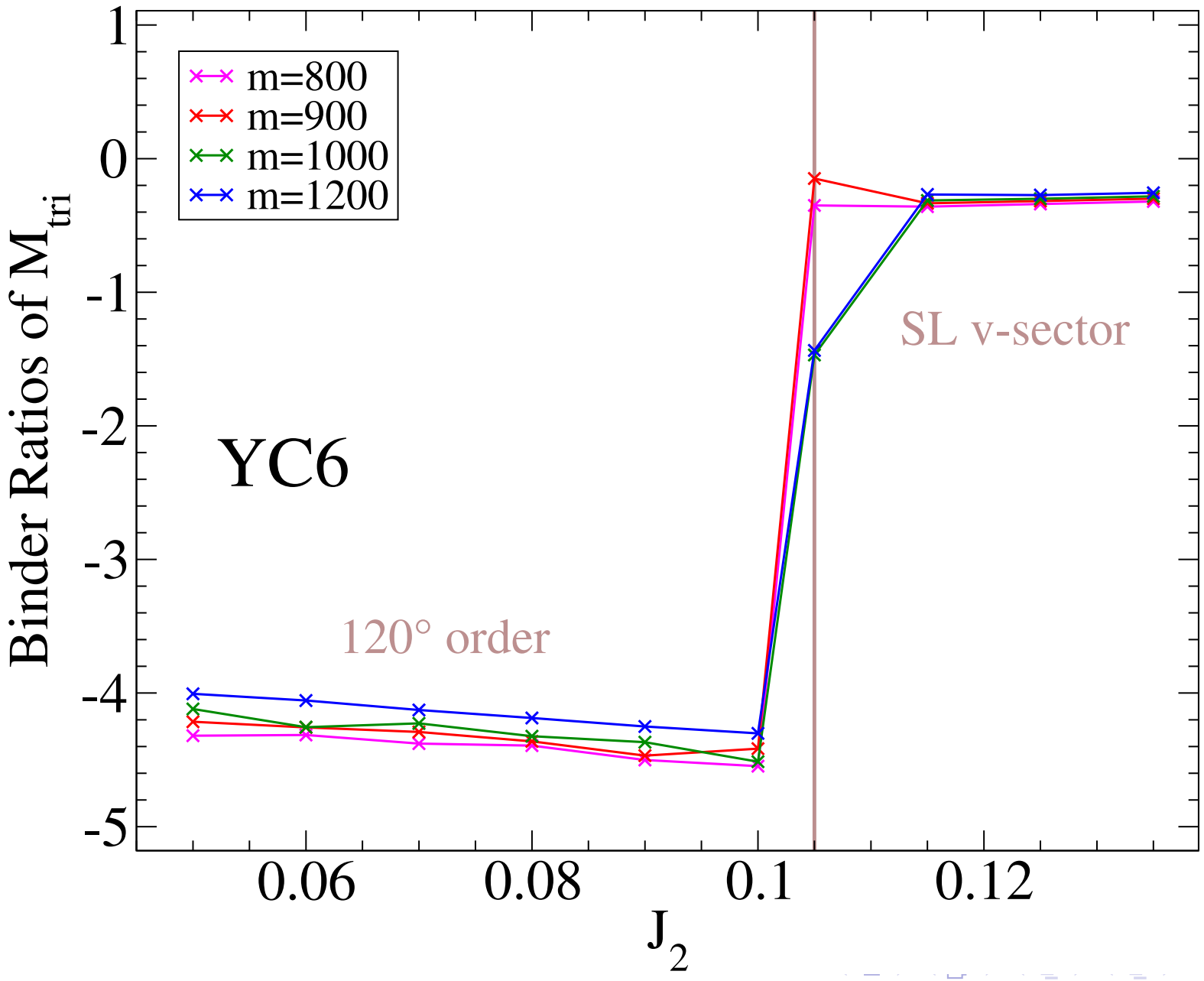
$$\langle M^4 \rangle_L = 3\kappa_2^2 L^2 + \kappa_4 L$$

Need L length scale – use correlation length ξ from transfer matrix

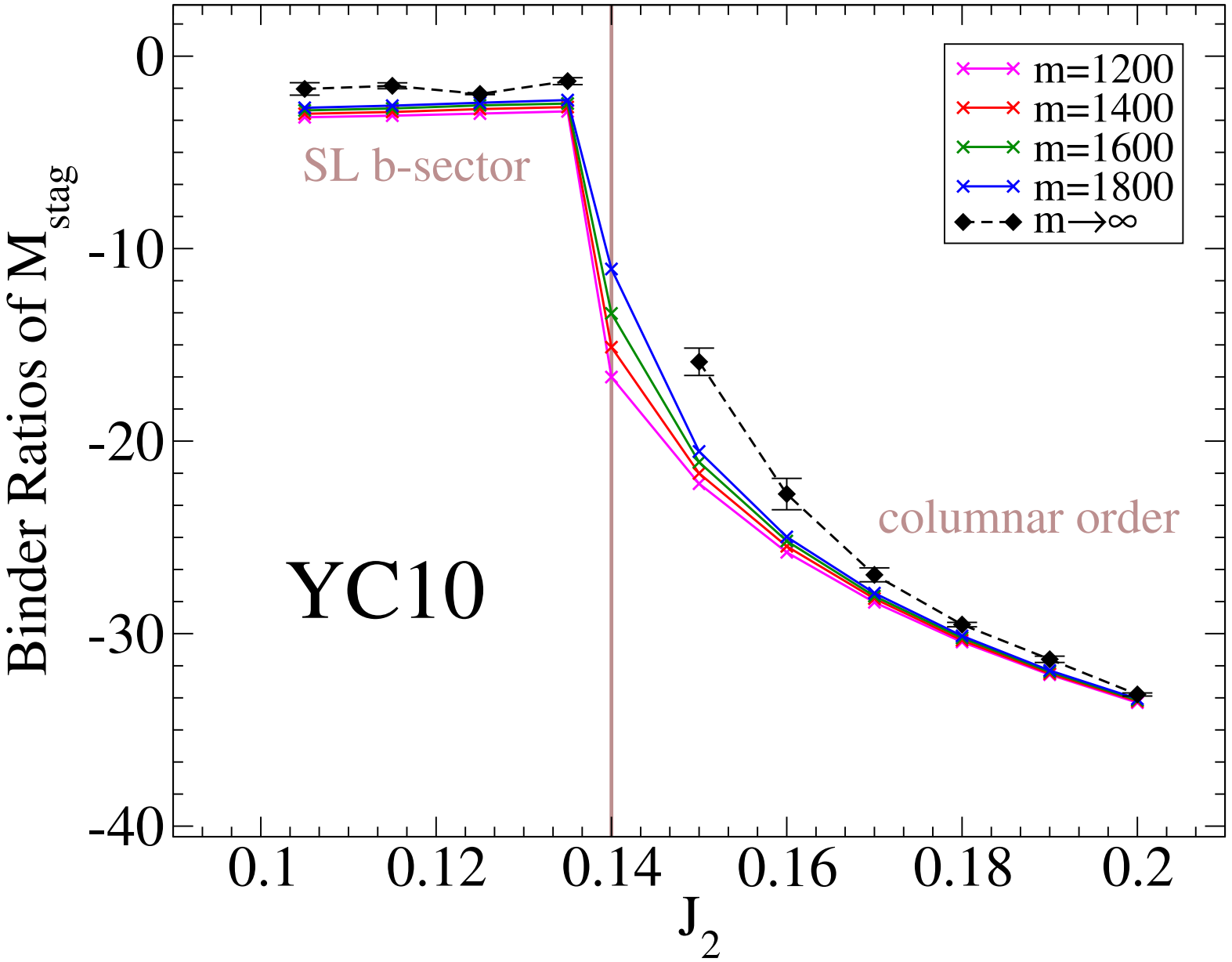
Binder ratio

$$\mathrm{BR} = \frac{\kappa_4}{\kappa_2^2 \xi}$$

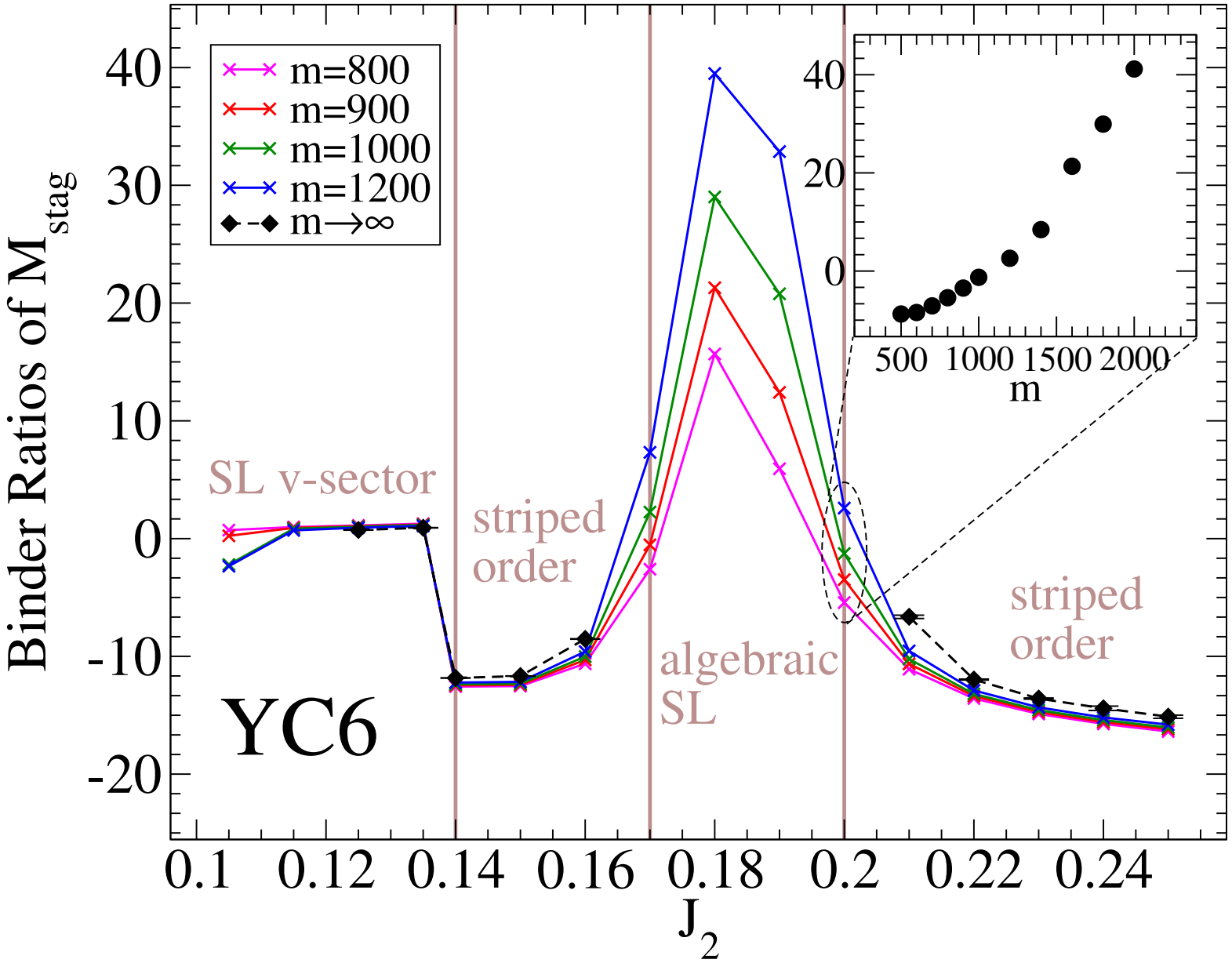
Binder ratio – 120° order



Binder ratio – staggered order

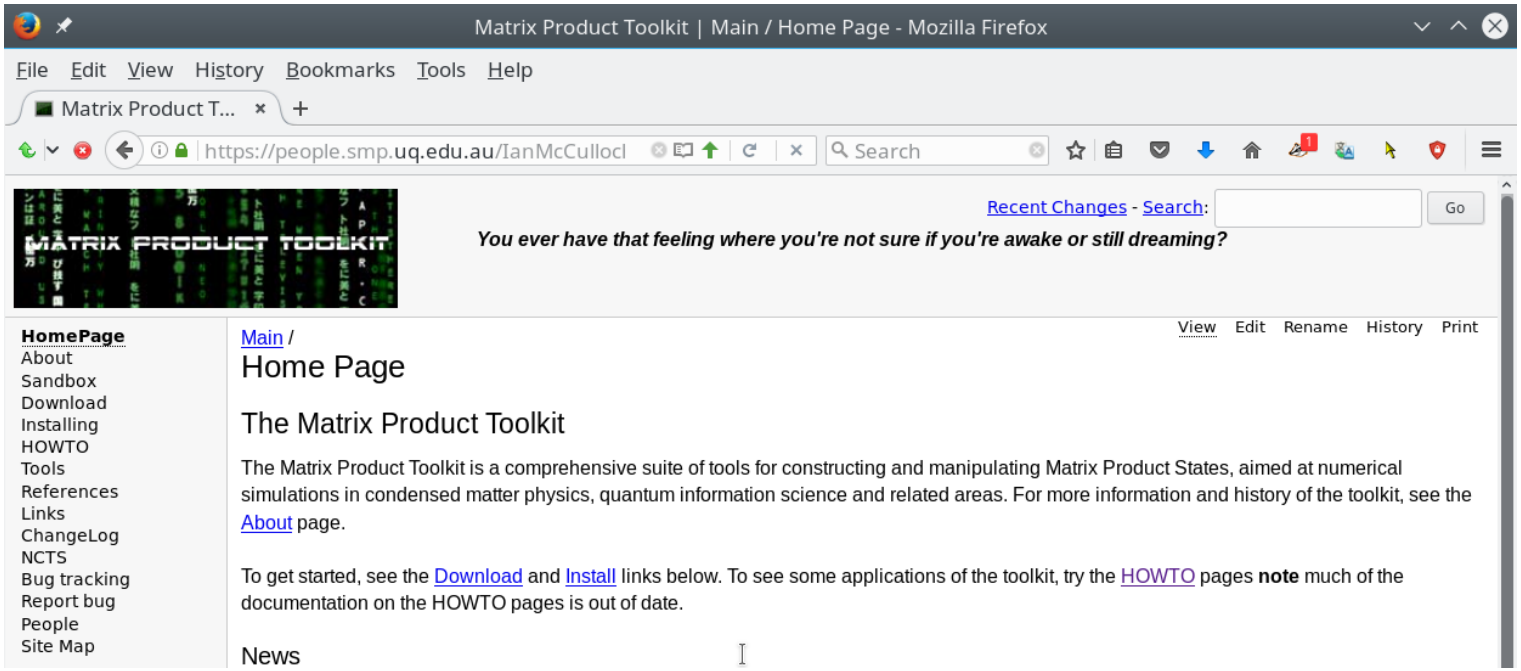


Binder ratio – staggered order YC6



Summary

Download the code: https://people.smp.uq.edu.au/IanMcCulloch/mptoolkit



- finite entanglement scaling
- higher moments
- Binder cumulant
- scaling functions
- symmetries of the entanglement spectrum
- Binder ratio for symmetric states