MATH4405
PROBLEM SHEET 1

1) Show that if $\mathcal{A}$ is an algebra of sets, and if, $\bigcup_n A_n$ belongs to $\mathcal{A}$ whenever $\{A_n\}$ is a sequence of disjoint sets in $\mathcal{A}$, then $\mathcal{A}$ is a σ-algebra.

2) Show that $\mathcal{B}(\mathbb{R})$ is generated by the collection of intervals $(-\infty, t]$ for which the end-point t is a rational number.

3) Show that if $\mathcal{A}$ is a σ-algebra in the Cartesian product $X_1 \times X_2$, then for every $x_1 \in X_1$ the collection $\{(A)_{x_1}; A \in \mathcal{A}\}$ is a σ-algebra on X_2. (Here $(A)_{x_1} = \{x \in X_2; (x_1, x) \in A\}$).

4) Find an infinite collection of subsets of $\mathbb{R}$ that contains $\mathbb{R}$, is closed under the formation of countable unions, and is closed under the formation of countable intersections, but is not a σ-algebra.

5) Show that $\mathcal{B}(\mathbb{R})$ is generated by the collection of all compact subsets of $\mathbb{R}$.