MATH1052 - PROJECT 1 SOLUTIONS

NOTE: For problems such as these there is not necessarily a unique way of solving the problem. The solutions given here are just one of the possible correct solutions.

PART A - 1.

```
>>ezsurf('x','x+2','z', [-3,0], [-15,5])
>>hold on
>>ezsurf('4-x^2-y^2', [-3,3])
```

2. Substituting (2) into (1) gives

$$z = 4x - 2x^2. \qquad (*)$$

Now

$$\frac{dz}{dx} = 4 - 4x, \qquad \frac{d^2z}{dx^2} = -4$$

from which we see that when $x = 1$ the first derivative is zero and since the second derivative is negative this corresponds to a local maximum. Substituting $x = 1$ into eq. (*) yields the maximum value of 2.

3.

```
>>ezplot('y-x-2', [-3,3])
>>hold on
>>ezplot('3-x^2-y^2')
>>ezplot('2-x^2-y^2')
>>ezplot('1-x^2-y^2')
```

PART B - 2.

$$\frac{\partial z}{\partial x} = -4x(x^2-1) - 2(2xy-1)(x^2y-x-1) \qquad (1)$$

$$\frac{\partial z}{\partial y} = -2x^2(x^2y-x-1) \qquad (2)$$

$$\frac{\partial^2 z}{\partial x^2} = -4(x^2-1) - 8x^2 - 4y(x^2y-x-1) - 2(2xy-1)^2 \qquad (3)$$

$$\frac{\partial^2 z}{\partial y^2} = -2x^4 \qquad (4)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -4x(x^2y-x-1) - 2x^2(2xy-1). \qquad (5)$$

From (2), we see that $\partial z/\partial y = 0$ when $x = 0$ or

$$x^2y - x - 1 = 0. \qquad (6)$$

Substituting $x = 0$ into (1) shows that $\partial z/\partial x = 2$ so there is no critical point with $x = 0$. Returning to (6) we find

$$y = \frac{x+1}{x^2}. \qquad (7)$$

Substituting (7) into (1) now yields

$$\frac{\partial z}{\partial x} = -4x(x^2 - 1)$$

so that $\partial z/\partial x = 0$ when $x = \pm 1$. We now substitute these values for x back into (7) to find the values 0,2 for y. Thus the critical points are (1,2) and (-1,0).

In order to classify these critical points we need to consider

$$D(x, y) = \frac{\partial^2 z}{\partial x^2} . \frac{\partial^2 z}{\partial y^2} - \left[\frac{\partial^2 z}{\partial x \partial y}\right]^2 .$$

Now

$$\begin{aligned}
\frac{\partial^2 z(1,2)}{\partial x^2} &= -26 \\
\frac{\partial^2 z(1,2)}{\partial y^2} &= -2 \\
\frac{\partial^2 z(1,2)}{\partial x \partial y} &= -6 \\
\frac{\partial^2 z(-1,0)}{\partial x^2} &= -10 \\
\frac{\partial^2 z(-1,0)}{\partial y^2} &= -2 \\
\frac{\partial^2 z(-1,0)}{\partial x \partial y} &= 2.
\end{aligned}$$

Thus

$$\begin{aligned}
D(1,2) &= 16 > 0 \ \text{ with } \ \frac{\partial^2 z(1,2)}{\partial x^2} < 0 \\
D(-1,0) &= 16 > 0 \ \text{ with } \ \frac{\partial^2 z(-1,0)}{\partial x^2} < 0
\end{aligned}$$

so both points are local maxima.

4.

Substituting $y = x + 1$ into $z(x, y)$ gives

$$\begin{aligned}
w(x) = z(x, x+1) &= -(x^2 - 1)^2 - (x^2(x+1) - x - 1)^2 \\
&= -(x^2 - 1)^2 - (x+1)^2(x^2 - 1)^2 \\
&= -(x^2 - 1)^2 \left[1 + (x+1)^2\right] \\
&= -(x^2 - 1)^2(x^2 + 2x + 2)
\end{aligned}$$

and we can calculate

$$\frac{dw}{dx} = -(x^2 - 1)^2(2x + 2) - 4x(x^2 - 1)(x^2 + 2x + 2).$$

Matlab commands for the full problem (including Matlab output)

```
>>ezplot('(x^2-1)^2+(x^2*y-x-1)^2-2', [-2,2], [-1,3])
>>hold on
>>ezplot('(x^2-1)^2+(x^2*y-x-1)^2-.5')
>>ezplot('(x^2-1)^2+(x^2*y-x-1)^2-.1')
>>ezplot('y-x-1')
>>x=fzero('-(x^2-1)^2* (2*x+2) -4*x*(x^2-1)*(x^2+2*x+2) , 0)
Zero found in the interval: [-0.32, 0.32].

x=


    0.2316


>> w=-(x^2-1)^2*(x^2+2*x+2)

w =

    -2.2541


>>ezplot('(x^2-1)^2+(x^2*y-x-1)^2-2.2541')
```

5.

Following the line $y = x + 1$ from (-1,0) to (1,2) the contour levels are decreasing as we move away from (-1,0) and increasing as we approach (1,2). The point where the contour lines crossover from decreasing to increasing in not a local minimum. The figure shows that at this point the contour line is tangential to the line $y = x + 1$. For any curve from (-1,0) to (1,2) it is possible for a similar situation to occur, which demonstates that it is *not necessary* for a local minimum to exist.