MATH1052 - MIDSEMESTER EXAM REVIEW

Motion in 3-dimensional space

- Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

- Find the velocity, acceleration and speed of the particle whose position vector is

$$\mathbf{r}(t) = e^t(\cos t\,\mathbf{i} + \sin t\,\mathbf{j}) + t\mathbf{k}.$$

Equation of a plane

- Find the equation of the plane that passes through the origin and the points (2,-4,6) and (5,1,3).

- Find the equation of the plane through the point (1,-1,1) and with normal vector $\mathbf{i} + \mathbf{j} - \mathbf{k}$.

Parameterization of curves and surfaces

- Show that if the point (a, b, c) lies on the hyperbolic paraboloid $z = y^2 - x^2$ then the lines with parametric equations

$$x = a + t, \quad y = b + t, \quad z = c + 2(b - a)t$$

and

$$x = a + t, \quad y = b - t, \quad z = c - 2(b + a)t$$

both lie entirely on this paraboloid.

- Give a parametrized expression for the curve of intersection for the surfaces

$$z = x^2 + 5,$$

and

$$z = 6x + y - 4.$$

Line integrals

- Find the length of the curve traced out by the position vector

$$\mathbf{r}(t) = \sqrt{2}\,t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$$

for $0 \leq t \leq 1$.

- Calculate the work done in moving a particle in a straight line from (1,1) to (4,-2) where the force is given by

$$\mathbf{F}(x, y) = \left(\frac{y^2}{x^2}\right)\mathbf{i} - \left(\frac{2y}{x}\right)\mathbf{j}.$$

Contour diagrams

- Draw contour diagrams for the values $f = 1, 0, -1$ for

$$f(x, y) = 2x - y + 1$$

and

$$f(x, y) = x^2 - 4x + y^2 + 2y.$$

Identify quadric surfaces

- Find an equation for the surface of all points that are equidistant from the point (-1,0,0) and the plane $x = 1$. Identify the surface.

- Reduce the following equation to standard form and identify the surface

$$x^2 + y^2 - 4z^2 + 4x - 6y - 8z = 13.$$

Limits

- Evaluate the following limits, if they exist, or show they do not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1},$$

$$\lim_{(x,y)\to(0,0)} x^2 \sin(x/y),$$

$$\lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^4}.$$

Partial derivatives

- For an ideal gas the pressure, volume and temperature are related through

$$PV = kT$$

where k is a constant. Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$

- For

$$u = x^5 y^4 - 3x^2 y^3 + 2x^2$$

show that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}.$$

Gradient

- For $f(x, y, z) = xy + yz^2 + xz^3$ find the gradient and evaluate it at the point (2,0,3).

- The temperature T in a metal ball is inversely proportional to the distance from the centre of the ball, which we take to be the origin. Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points towards the origin.

Conservative fields

- Find a function $\phi(x, y, z)$ such that $\mathbf{F} = \nabla\phi$ where

$$\mathbf{F} = (2xz + \sin y)\mathbf{i} + x\cos y\mathbf{j} + x^2\mathbf{k}.$$

- Show that

$$\mathbf{F}(x, y) = x\exp y\mathbf{i} + y\exp x\mathbf{j}$$

is not conservative.

Directional derivative

- Find the directional derivative of

$$f(x, y, z) = \frac{x}{y + z}$$

at the point (4,1,1) in the direction of the vector

$$\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}.$$

- Find the directional derivative of

$$f(x, y) = x^2y^3 + 2x^4y$$

at the point (1,-2) in the direction with angle $\pi/3$ above the positive x-axis.

Differentials

- Find the differential of the function

$$w = x\sin yz.$$

- Use differentials to estimate the volume of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

Approximations to surfaces

- Find the equation of the tangent plane to the surface

$$f(x,y) = \ln(x^2 - y^3)$$

at the point $x = 1$, $y = -1$.

- Find the second order Taylor's series approximation for the function

$$f(x,y) = x^2 - xy + y^2$$

at the point $x = y = 0$.

Find and classify critical points

- Find and classify the critical points for the function

$$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2.$$

- Find the dimensions of the rectangular box with largest volume if the total surface area is given as 64 cm^2.