Phase-dependent correlations and spectra of intense squeezed light in four-wave mixing above threshold

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(Submitted 11 May 1993; resubmitted 21 January 1994)
Zh. Eksp. Teor. Fiz. **105**, 1161–1180 (May 1994)

The spectral, correlation, and noise properties of squeezed light generated as a result of intracavity four-wave mixing (FWM) above threshold under the influence of two pump fields of different frequencies are studied. The effects of quantum fluctuations in the intensity spectra and the spectra of intensity fluctuations of the cavity-output modes of radiation, as well as the two-time correlation functions of fluctuations of the photon numbers and phases of the intracavity modes, are considered. Spectra of fluctuations of the difference and sum of the quadrature components of pairs of correlated modes as applied to measurement of the quantum noise level by the twin-homodyning method are studied. As a result, squeezing of fluctuations in the difference and sum of the quadratures, which determine the phase and amplitude fluctuations, respectively, is found. It is shown that the squeezing effects in these cases are caused by a positive correlation between the phase fluctuations of the modes, or by a negative correlation between their intensity fluctuations.

1. INTRODUCTION

One direction taken recently by optical studies which deserves special attention is the theoretical analysis and experimental realization of efficient schemes for producing strong optical fields in quantum states, in particular squeezed light. In optical circuits based on the interaction of electromagnetic modes in a nonlinear medium in a cavity this problem often rests on the possibility of analyzing the quantum fluctuations and the effects of mode correlations in the above-threshold generation regime. Up to the present time this analysis has been carried out in most detail for processes such as nondegenerate parametric generation of light and four-wave mixing in a monochromatic pump field (see, e.g., Refs. 1–6 and the earlier work cited there). The results relate in particular to the spectra of squeezed fluctuations of two-mode quadrature components,[1,4] fluctuations in the sum or difference of the intensities,[1,2,5–8] and to the phase sums of the radiation fields of two correlated modes.[1,5,9]

Kryuchkyan and Kheruntsyan[10,11] have proposed a difference scheme for obtaining intense nonclassical light, based on four-wave mixing in an optical cavity subjected to two laser fields with different frequencies. In this process the two pump modes with frequencies ω_1 and ω_2 lead to the formation of a signal mode with frequency ω_0 such that $\omega_1+\omega_2=2\omega_0$. The main advantage of this process in comparison with nondegenerate parametric light generation and four-wave mixing is that it is possible to produce intense single-mode squeezed light near the frequencies of each of the modes ω_0, ω_1, and ω_2 in the above-threshold regime. This means that the proposed scheme is a possible way to produce a laser without population inversion for squeezed light. The process also gives rise to a different nonclassical effect, the suppression of quantum fluctuations in the sum of the intensities of the ω_1 and ω_2 modes below the corresponding vacuum level.[12]

This work is a continuation of our previous studies[10–12] and is aimed at further investigating the properties of nonclassical light in four-wave mixing in two laser fields. In this paper two sets of questions are treated. In one of them, which relates to the "conventional" design of experiments, nonclassical effects in optical spectra are studied. With this in mind we have calculated the intensity spectra (Sec. 3) and the spectra of the intensity fluctuations (Sec. 4) for each of the modes ω_0, ω_1, and ω_2 at the output from the cavity. We show that the intensity spectra together with a δ-function peak corresponding to the coherent part of the radiation contain all the spectrally broadened incoherent parts produced by the quantum fluctuations of the photon number and phases of the three modes. When the time correlations of these fluctuations have an oscillatory nature the incoherent part of the spectrum acquires a structure with four peaks. Under these same conditions, however, the intensity fluctuation spectra have two peaks.

Another set of questions relates to the treatment of the quantum-statistical properties of light due to correlations in phase and amplitude or intensity between the modes ω_i and ω_j $(i,j=0,1,2)$. We have investigated the suppression of the quantum fluctuations of the sum or difference of phase-dependent quadrature components for pairs of correlated modes ω_i,ω_j below the corresponding vacuum level (Secs. 5–8). These questions are considered in connection with the recently devised twin-homodyning experimental scheme with two reference waves (see, e.g., Refs. 13 and 14).

It is appropriate now to mention the fundamental properties of four-wave mixing in two laser fields which stem from the new results in the present work. One of these is that in the above-threshold regime, on account of the absence of phase diffusion, the quantum-statistical averages of the amplitudes of the light fields at the output from the cavity near each of the frequencies ω_j have definite phases and are nonzero. It follows, in particular, that single-mode

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squeezed coherent states arise in this process. In the scheme for measuring the fluctuations of the sum or difference of the field quadratures, this property implies that for certain fixed ratios of the phases of the fields and reference waves these fluctuations will be manifested either through phase fluctuations or intensity fluctuations of the modes.

For comparison let us recall that in nondegenerate parametric light generation and four-wave mixing in the above-threshold regime only the sum of the phases of the signal and parasitic modes is determined, while the phases of each of these modes separately are determined through the diffusion of the phase difference.[1,5,15] This fact restricts the possibility of studying phase-dependent nonclassical effects for these systems, e.g., single-mode squeezed states. In addition, this implies that fluctuations of the sum or difference of the components in quadrature are determined by the contributions of the phase fluctuations and mode intensities simultaneously for any choice of the reference waves.[1,2]

As a consequence, in the present work the following quantum fluctuation suppression effects are found. For the case in which the phase fluctuations of the fields dominate, we find that fluctuations of the quadrature difference between the pump modes ω_1 and ω_2 are squeezed, along with those between the ω_0 signal mode and both of the other modes ω_1 and ω_2 (Secs. 5 and 7). This effect is close to 100% squeezing and is a consequence of the strong positive correlation between the phase fluctuations of the modes.

When the amplitude fluctuations dominate, squeezing occurs for the sum of the quadrature components. This is caused by the negative correlation between the intensity fluctuations of these modes (Sec. 8).

2. NONLINEAR SYSTEM AND LINEARIZED EQUATIONS OF MOTION

This system is based on the following phenomenological model of parametric four-wave interaction in a cavity acted on by two laser fields. The nonlinear medium, described by the third-order susceptibility $\chi^{(3)}$, is in an annular cavity with eigenfrequencies $\omega_0,\omega_1,\omega_2$. Collinear mixing takes place with pump modes ω_1 and ω_2 and signal modes with frequency ω_0 such that $\omega_1+\omega_2=2\omega_0$, together with the resonance condition $\mathbf{k}_1+\mathbf{k}_2=2\mathbf{k}_0$ between the wave vectors of the modes. The pump modes are perturbed by two external coherent fields with frequencies ω_1 and ω_2 such that the mode ω_0 is excited spontaneously. Attenuation of the three modes on account of the cavity mirror is taken into account, and for simplicity we neglect the detuning of the cavity. This system can be described by the Hamiltonian

$$H=\sum_{j=0}^{2}\hbar\omega_j a_j^+ a_j + i\hbar\frac{\chi}{2}(a_1a_2a_0^{+2}-a_1^+a_2^+a_0^2) + i\hbar\sum_{k=1}^{2}(E_k e^{-i\omega_k t}a_k^+ - E_k^* e^{i\omega_k t}a_k) + \sum_{j=0}^{2}(a_j^+\Gamma_j + a_j\Gamma_j^+), \quad (1)$$

where a_j^+, a_j are the creation and annihilation operators of the three modes ω_j $(j=0,1,2)$ in the cavity, $\chi/2$ is the effective coupling constant, $E_{1,2}$ are the complex amplitudes of the perturbing fields in the cavity, and the reservoir operators Γ_j^+,Γ_j describe the damping of the modes ω_j in the cavity with damping rates or cavity linewidths γ_j respectively.[1)]

In our previous work[10] we used the Fokker–Planck method developed by Drummond and Gardiner[18] in the positive P representation to derive random equations of motion for the c-number functions α_j^+,α_j corresponding to the slowly varying time-dependent operators a_j^+, a_j, and equations for the quantities

$$n_j=\alpha_j^+\alpha_j, \quad \psi_j=\frac{1}{2i}\ln(\alpha_j/\alpha_j^+), \quad (2)$$

describing the intensities (in units of photon number) and phases of the modes ω_j. The remaining analysis of the quantum fluctuation effects of the radiation modes in the above-threshold regime is carried out by linearizing the equations of motion for the quantity $n_j(t)$ and $\psi_j(t)$ by introducing small perturbations (fluctuations)

$$\delta n_j(t)=n_j(t)-n_j^0, \quad \delta\psi_j(t)=\psi_j(t)-\psi_j^0 \quad (3)$$

about the corresponding time-independent quantities n_j^0 and ψ_j^0.

The systems of linearized stochastic equations of motion for the Fourier components $\delta n_j(\omega)$, $\delta\psi_j(\omega)$ of the fluctuations $\delta n_j(t)$ in the intensity and $\delta\psi_j(t)$ in the phases for the modes ω_j in matrix form finally become[10]

$$(A-i\omega I)\delta n(\omega)=F(\omega), \quad (4)$$

$$(\bar{A}-i\omega I)\delta\psi(\omega)=f(\omega). \quad (5)$$

Here the matrices A and $\bar{A}$ are equal to

$$A=\begin{pmatrix} 0 & -\frac{\gamma_0 n_0^0}{n_1^0} & -\frac{\gamma_0 n_0^0}{n_2^0} \\ \gamma_0 & \gamma & \frac{\gamma_0 n_0^0}{2n_2^0} \\ \gamma_0 & \frac{\gamma_0 n_0^0}{2n_1^0} & \gamma \end{pmatrix},$$

$$\bar{A}=\begin{pmatrix} 2\gamma & -\gamma_0 & -\gamma_0 \\ \frac{\gamma_0 n_0^0}{n_1^0} & \gamma & -\frac{\gamma_0 n_0^0}{2n_1^0} \\ \frac{\gamma_0 n_0^0}{n_2^0} & -\frac{\gamma_0 n_0^0}{2n_2^0} & \gamma \end{pmatrix} \quad (6)$$

I is the unit matrix, $\gamma\equiv\gamma_1=\gamma_2$ is the cavity width, assumed to have the same value for the pump modes ω_1 and ω_2, and δn, $\delta\psi$, F, and f denote column vectors of the form

$x=(x_0,x_1,x_2)^T$. The quantities F_j and f_j are zero-mean Gaussian noise terms and the following nonzero autocorrelations:

$$\langle F_0(\omega)F_0(\omega')\rangle=2\gamma_0 n_0^0\delta(\omega+\omega'),$$

$$\langle F_1(\omega)F_2(\omega')\rangle=-\gamma_0 n_0^0\delta(\omega+\omega'), \tag{7}$$

$$\langle f_0(\omega)f_0(\omega')\rangle=-\frac{\gamma_0}{2n_0^0}\delta(\omega+\omega'),$$

$$\langle f_1(\omega)f_2(\omega')\rangle=\frac{\chi^2 n_0^0}{4\gamma_0}\delta(\omega+\omega'). \tag{8}$$

We note also that the results given here and in what follows reflect the case of equal amplitudes and arbitrary phases for the perturbing fields: $E_{1,2}=E\exp(i\Phi_{1,2})$.

We also introduce the results (needed in what follows) for the quantities n_j^0 and ψ_j^0 in the above-threshold generation regime for $\varepsilon\equiv E/E_{\rm th}>1$, where $E_{\rm th}=\gamma(\gamma_0/\chi)^{1/2}$ is the threshold value of the amplitudes E of the perturbing fields. In this regime we must distinguish two types of stationary solution.

The first of these is stable in the range $1<\varepsilon<2$ and takes the form

$$n_0^0=2\gamma(\varepsilon-1)/\chi,\quad n_1^0=n_2^0=\gamma_0/\chi, \tag{9}$$

$$\psi_0^0=(\psi_1^0+\psi_2^0)/2,\quad \psi_1^0=\Phi_1,\quad \psi_2^0=\Phi_2, \tag{10}$$

where $\Phi_{1,2}$ are the phases of the perturbing fields $E_{1,2}$.

The other solutions are stable in the region $\varepsilon>2$ and have bistable behavior:

$$n_0^0=2\gamma/\chi, \tag{11}$$

$$n_1^0=\frac{\gamma_0}{2\chi}(\varepsilon^2-2+\varepsilon\sqrt{\varepsilon^2-4}),$$

$$n_2^0=\frac{\gamma_0}{2\chi}(\varepsilon^2-2-\varepsilon\sqrt{\varepsilon^2-4}), \tag{11a}$$

or

$$n_1^0=\frac{\gamma_0}{2\chi}(\varepsilon^2-2-\varepsilon\sqrt{\varepsilon^2-4}),$$

$$n_2^0=\frac{\gamma_0}{2\chi}(\varepsilon^2-2+\varepsilon\sqrt{\varepsilon^2-4}). \tag{11b}$$

Here the stationary values of the phases ψ_j^0 are the same for the solutions (11a) and (11b) and are identical with (10).

The values $\varepsilon=1$, 2 are instability points of the system, since the time-independent solutions there are unstable.

3. INTENSITY SPECTRA AND CORRELATION FUNCTIONS OF THE FLUCTUATIONS

Let us consider the spectra of the light intensity at the output of the cavity near each of the modes ω_j. These are given by the expression

$$N_j(\omega)=\int_{-\infty}^{+\infty}d\tau\exp[i(\omega-\omega_j)\tau]\langle b_j^+(t)b_j(t+\tau)\rangle, \tag{12}$$

where b_j are the field amplitude operators at the output from the cavity near ω_j. For the case we are considering, in which the input and output radiation are obtained at one of the mirrors of the annular cavity, they are related to the corresponding operators c_j at input and the operators a_j as follows (see, e.g., Ref. 19):

$$b_j=\sqrt{2\gamma_j}a_j-c_j,\quad (j=0,1,2),$$

$$E_k=\sqrt{2\gamma_k}\langle c_k\rangle,\quad (k=1,2), \tag{13}$$

where the corresponding commutators are

$$[b_i(t),b_j^+(t')]=[c_i(t),c_j^+(t')]=\delta_{ij}\delta(t-t').$$

The next stage of the calculations using Eqs. (12) and (13) is carried out using the correspondence[18,20] between the normal-ordered averages of the operators a_j^+,a_j and the averages in the P representation of the numerical functions α_j^+,α_j, along with the δ-correlation properties of the noise terms. After elementary manipulations we find to lowest order in the fluctuations

$$N_j(\omega)=2\pi|b_j^0|^2\delta(\Omega)+\frac{\gamma_j}{2n_j^0}\langle\delta n_j(-\Omega)\delta n_j(\Omega)\rangle$$

$$+2\gamma_j n_j^0\langle\delta\psi_j(-\Omega)\delta\psi_j(\Omega)\rangle. \tag{14}$$

Here we have written $\Omega=\omega-\omega_j$ and b_j^0 are the coherent component of the field amplitudes at the output of the cavity, equal to

$$b_0^0=\sqrt{2\gamma_0}\alpha_0^0,\quad b_{1,2}^0=\sqrt{2\gamma}\alpha_{1,2}^0-\langle c_{1,2}\rangle, \tag{15}$$

where $\alpha_j^0=(n_j^0)^{1/2}\exp(i\psi_j^0)$.

Thus, the intensity spectra near the frequencies $\omega=\omega_j$ contain a narrow δ-function peak corresponding to the coherent contribution. The integrated intensities of the peaks or the average photon numbers per unit time of each mode at the output of the cavity, calculated in the semiclassical approximation using Eqs. (9) and (11) in the range $1<\varepsilon<2$, are

$$|b_0^0|^2=4\gamma_0\gamma(\varepsilon-1)/\chi,$$

$$|b_1^0|^2=|b_2^0|^2=\frac{2\gamma_0\gamma(1-\varepsilon/2)^2}{\chi} \tag{16}$$

and in the region $\varepsilon>2$ are equal to

$$|b_0^0|^2=4\gamma_0\gamma/\chi,\quad |b_1^0|^2=|b_2^0|^2=\frac{2\gamma_0\gamma(\varepsilon^2/4-1)}{\chi}. \tag{17}$$

The spectral broadening is due to the quantum fluctuations of the radiation field, and in lowest order is determined by the correlation functions of the second-order fluctuations of the photon number and phases of the modes ω_j.

We turn now to the evaluation of these quantities. Using the solutions of Eqs. (4) and the noise correlations (7), we find after some algebraic manipulations

$$\frac{\gamma_0}{2n_0^0}\langle\delta n_0(-\Omega)\delta n_0(\Omega)\rangle$$

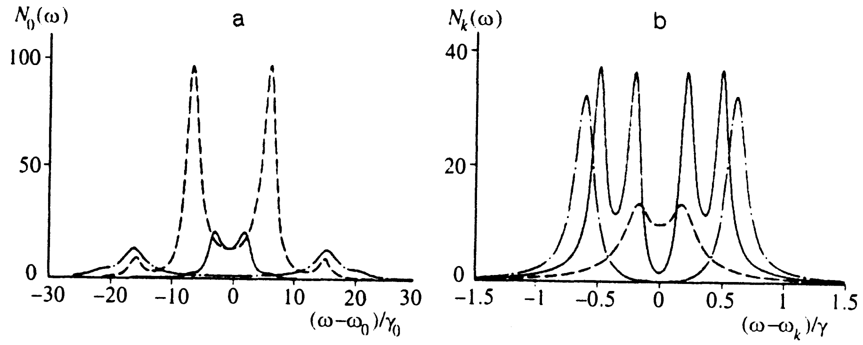

FIG. 1. a) Intensity spectrum $N_0(\omega)$ of the signal mode as a function of $(\omega-\omega_0)/\gamma_0$ for $\varepsilon=2.2$, $r=0.1$ (solid); $\varepsilon=2.2$, $r=0.02$ (dashed); and $\varepsilon=3$, $r=0.02$ (chain curve). b) Spectra of the intensity $N_k(\omega)$, $(k=1,2)$ for the pump modes ω_1 and ω_2 as a function of $(\omega-\omega_k)/\gamma$. The spectral curves coincide in the region $1<\varepsilon<2$, where $n_1^0=n_2^0$ holds, and differ from one another in the region $\varepsilon>2$, where $n_1^0\neq n_2^0$ holds. The curves shown for the region $\varepsilon>2$ correspond to a pump wave which drops off in intensity, with the parameters $\varepsilon=1.8$, $r=0.01$ (dashed); $\varepsilon=2.2$, $r=0.05$ (solid); and $\varepsilon=6$, $r=0.01$ (chain curve).

$$=\frac{1}{d(\Omega)}\{r^4(\Omega/\gamma_0)^4+2r^2(1-q)(\Omega/\gamma_0)^2 +(1-q)^2+4q(4p-q-1)\}, \tag{18a}$$

$$\frac{\gamma}{2n_k^0}\langle\delta n_k(-\Omega)\delta n_k(\Omega)\rangle$$

$$=\frac{2q_k}{r^2d(\Omega)}\{[r^2-2rq_n+q_n](\Omega/\gamma)^2 +r^2(1-q_n)^2+4r^2q_n^2\},\quad (n\neq k=1,2). \tag{18b}$$

In these expressions the quantity $d(\Omega)$, defined as the determinant of the matrix $A-i\Omega I$, is equal to

$$d(\Omega)=|\det(A-i\Omega I)|^2/(r^2\gamma^6)$$
$$=4[r(\Omega/\gamma_0)^2-2q-p]^2+(\Omega/\gamma_0)^2[r^2(\Omega/\gamma_0)^2 +q-2rp-1]^2, \tag{19}$$

where we have used the notation

$$r=\gamma_0/\gamma,\quad q_{1,2}=rn_0^0/(2n_{1,2}^0),\quad p=q_1+q_2,\quad q=q_1q_2. \tag{20}$$

The parameters $q_{1,2}$, p, and q are different in the generation regimes (1) and (2), where they are respectively equal to

$$(1)\quad 1<\varepsilon<2:\quad q_1=q_2=\varepsilon-1; \tag{21}$$

$$(2)\quad \varepsilon>2:\quad q_{1,2}=2/(\varepsilon^2-2\pm\varepsilon\sqrt{\varepsilon^2-4}) \tag{22a}$$

for the solutions (11a) and

$$q_{1,2}=2/(\varepsilon^2-2\mp\varepsilon\sqrt{\varepsilon^2-4}) \tag{22b}$$

for the solutions (11b), where for both cases (11a) and (11b) we have

$$p=\varepsilon^2-2,\quad q=1,\quad (\varepsilon>2). \tag{23}$$

Note also that at "zero" frequency, the quantity $d(0)$ is equal to

$$d(0)=4(p-2q)^2=\begin{cases}16(\varepsilon-1)^2(2-\varepsilon)^2, & 1<\varepsilon<2,\\ 4(\varepsilon^2-4)^2, & \varepsilon>2,\end{cases} \tag{24}$$

i.e., at the frequencies $\omega=\omega_j$ and at the instability points $\varepsilon=1,2$, the averages (18) diverge, the magnitude of the quantum fluctuations grows without bound, and the results of the linearized theory become inapplicable.

The averages to second order in the phase fluctuations, which, as shown in Ref. 10, determine the minimum spectra for squeezing of the quadrature components of the modes ω_j, are[2)]

$$2\gamma_0n_0^0\langle\delta\psi_0(-\Omega)\delta\psi_0(\Omega)\rangle$$
$$=-\frac{1}{\bar{d}(\Omega)}\{r^4(\Omega/\gamma_0)^4+2r^2(1-q)(\Omega/\gamma_0)^2 +(1-q)^2-4q\varepsilon^2\}, \tag{25a}$$

$$2\gamma n_k^0\langle\delta\psi_k(-\Omega)\delta\psi_k(\Omega)\rangle$$
$$=-\frac{2}{r^2\bar{d}(\Omega)}\{[q_kr^2-q(1+2r)](\Omega/\gamma)^2 +r^2q_k(1+q_n)^2\},\quad (n\neq k=1,2), \tag{25b}$$

where

$$\bar{d}(\Omega)=|\det(\bar{A}-i\Omega I)|^2/(r^2\gamma^6)$$
$$=4[(r^2+r)(\Omega/\gamma_0)^2-\varepsilon^2]^2+(\Omega/\gamma_0)^2[r^2(\Omega/\gamma_0)^2 +q-4r-2rp-1]^2, \tag{26}$$

and $\bar{d}(0)=4\varepsilon^2$ throughout the entire above-threshold region $\varepsilon>1$.

We mention here the results of studying the intensities $N_j(\omega)$ of the modes ω_j in the absence of the coherent components $\sim\delta(\omega-\omega_j)$. Analysis of Eqs. (14), (18), (19), (25), and (26) shows that in the range $1<\varepsilon<2$ and for $r\ll1$, the spectra take the form of a single peak at the central carrier frequency ω_j, whose height depends only on ε and whose width is determined by the ratio $r=\gamma_0/\gamma$. As r increases, the spectra broaden and in addition to the central peak two symmetric peaks at sideband frequencies develop, the height and location of which are determined by the parameters ε and r. In the region $\varepsilon>2$ the form of the intensity spectra becomes more complicated. Depending on the values of ε and r, which determine the magnitudes of the contributions of the phase and photon-number fluctuations in the expression (14) for the spectrum, two or four peaks can develop, located symmetrically with respect to the central frequency ω_j. Figure 1 shows a plot illustrating this with the intensity of the signal mode and one of the pump modes.

For a more detailed analysis of the situation we should keep in mind that the broadening and structure of the spectra are determined by the contributions of the Fourier components of the two-time correlation functions $\langle\delta n_j(t)\delta n_j(t+\tau)\rangle$, $\langle\delta\psi_j(t)\delta\psi_j(t+\tau)\rangle$, and depend on the nature of the correlations of the instantaneous fluctuations. From the results of the calculations given in the Appendix [see Eqs. (A1) and (A2)] we see that the τ dependence of these correlation functions is given respectively by the exponential factors $\exp(-\lambda_m|\tau|)$ and $\exp(-\bar{\lambda}_m|\tau|)$, where λ_m, $\bar{\lambda}_m$ $(m=1,2,3)$ are the eigenvalues of the matrices A and $\bar{A}$ given in Eq. (6). For the parameters ε and r, for which $\lambda_{1,2}$ and $\bar{\lambda}_{1,2}$ are real (note that λ_3 and $\bar{\lambda}_3$ are real for arbitrary ε and r) the fluctuation correlations die out monotonically with increasing τ, and the spectra have the same form, with a width given by the damping rate of the correlations. Splitting of the spectra occurs for values of ε and r that yield complex values $\lambda_{1,2}$ and $\bar{\lambda}_{1,2}$. In this case, as shown in the Appendix, the correlation functions are

$$\langle\delta n_j(t)\delta n_j(t+\tau)\rangle =2[K'_{\beta\beta}\cos(\lambda''|\tau|)+K''_{\beta\beta}\sin(\lambda''|\tau|)] \times\exp(-\lambda'|\tau|)+K_{3,\beta\beta}\exp(-\lambda_3|\tau|), \qquad (27)$$

$$\langle\delta\psi_j(t)\delta\psi_j(t+\tau)\rangle =2[\bar{K}'_{\beta\beta}\cos(\bar{\lambda}''|\tau|)+\bar{K}''_{\beta\beta}\sin(\bar{\lambda}''|\tau|)] \times\exp(-\bar{\lambda}'|\tau|)+\bar{K}_{3,\beta\beta}\exp(-\bar{\lambda}_3|\tau|), \qquad (28)$$

where we have written $\beta=j+1$, $\lambda'=\mathrm{Re}\,\lambda_{1,2}$, $\bar{\lambda}'=\mathrm{Re}\,\bar{\lambda}_{1,2}$, and they behave as damped oscillations whose frequencies are determined by the imaginary parts of the eigenvalues $\lambda''=\mathrm{Im}\,\lambda_1=-\mathrm{Im}\,\lambda_2$, $\bar{\lambda}''=\mathrm{Im}\,\bar{\lambda}_1=-\mathrm{Im}\,\bar{\lambda}_2$. In this case the multipeak structure of the spectra is essentially the beat spectrum of the two-time correlation functions of the photon-number and phase fluctuations of the radiation fields, and the locations of the side peaks are given by the frequencies $\omega_j\pm\lambda''$, $\omega_j\pm\bar{\lambda}''$.

4. SPECTRA OF THE INTENSITY FLUCTUATIONS

The spectra of the fluctuations of the field intensities at the output of the resonator for each of the modes ω_j are given by

$$P_j(\omega)=\int_{-\infty}^{+\infty} d\tau\, e^{i\omega\tau}\langle b_j^+(t)b_j(t),b_j^+(t+\tau)b_j(t+\tau)\rangle, \qquad (29)$$

in which we have written $\langle A,B\rangle=\langle AB\rangle-\langle A\rangle\langle B\rangle$. Going over to normal ordering and then to a random variable, we can show without difficulty that to lowest order in the fluctuations the spectrum (29) can be written

$$P_j(\omega)=|b_j^0|^2\left[1+\frac{2\gamma_j}{n_j^0}\langle\delta n_j(-\omega)\delta n_j(\omega)\rangle\right]. \qquad (30)$$

The first terms in these expressions are the quantum fluctuation level for the coherent fields. The nontrivial noise terms are determined by the correlations of the photon-number fluctuations; unlike the intensity spectra (14), they do not contain phase correlation functions. This difference is to be expected. We recall that for Gaussian fields the information content of the intensity spectrum and the spectrum of the corresponding noise is essentially the same. It is clear, however, that for nonclassical light this situation does not hold in general.

Analysis of the intensity fluctuation spectra found using Eqs. (18) and (30) for $1<\varepsilon<2$ and for $r\ll1$ reveals that the spectra have the same shape with a maximum at zero frequency, $\omega=0$. As r increases, two symmetric auxiliary peaks develop. Their height increases as a function of r, and they move away from $\omega=0$ for the pump modes and approach $\omega=0$ for the ω_0 mode. In the region $\varepsilon>2$, the split structure of the spectrum can be seen both for $r\gg1$ and for $r\ll1$.

A more detailed treatment of the spectra, analogous to that carried out in Sec. 3, requires recourse to two-time correlation functions for the photon-number fluctuations (27) [see Eq. (A1)]. This enables us to explain, in particular, the multipeak structure of the spectra. In the region $\varepsilon>2$, however, it is necessary to use the expressions (A7) for the eigenvalues λ_m obtained explicitly by using the approximations $\varepsilon^2\gg1$ and $r\varepsilon^2\gg1$. Consequently, the quantitative analysis of the temporal solutions and explanations of the shapes of the spectral curves using these results are inapplicable when these conditions are violated.[3)]

When we compare the quantitative results for the spectra of the intensities $N_j(\omega)$ and their fluctuations $P_j(\omega)/|b_j^0|^2$, the following circumstance should be noted. As can be seen from the expressions for the eigenvalues λ_m and $\bar{\lambda}_m$ and the results (A1) and (A2), the damping rates of the correlations of the photon-number and phase fluctuations for $\gamma_0\ll\gamma$ are first-order quantities. Hence the contributions of these fluctuations to the broadened structure of the intensity spectra are comparable. The final shapes of the $N_j(\omega)$ and $P_j(\omega)/|b_j^0|^2$ spectra in this region differ considerably. In the other limit $\gamma_0\gg\gamma$ the damping of the phase fluctuation correlations is determined by the width γ_0, and takes place much more rapidly than the damping of the correlations of the photon-number fluctuations, determined by the width γ. In this case the phase fluctuations therefore make no contribution to the intensity spectrum: the shapes of the $N_j(\omega)$ and $P_j(\omega)/|b_j^0|^2$ spectral curves are approximately the same [to within a scale factor of 4 and disregarding the coherent δ-function components in $N_j(\omega)$]. Plots of the fluctuation spectra showing this behavior are given in Fig. 2.

5. CORRELATIONS OF THE PHASE AND PHOTON-NUMBER FLUCTUATIONS OF THE MODES IN THE CAVITY

Let us turn now to a discussion of the statistical and noise properties of the radiation field of the system we are considering, caused by the effects of intermode correlations. In the present section we treat these effects for discrete radiation modes in the cavity. The following sections are devoted to studying the corresponding spectra for light at the output of the cavity.

Usually one of the most difficult questions to treat theoretically is that of the quantum fluctuations of the phases

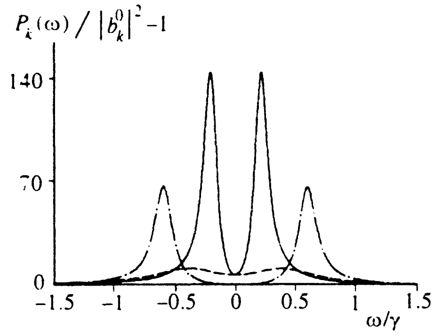

FIG. 2. Intensity fluctuation spectrum $P_k(\omega)/|b_k^0|^2$ for the pump modes ω_k $(k=1,2)$ as a function of ω/γ. The curves shown correspond to a pump mode decreasing in intensity with the following values of the parameters: $\varepsilon=2.2$, $r=0.05$ (solid); $\varepsilon=2.2$, $r=0.2$ (dashed); and $\varepsilon=6$, $r=0.01$ (chain curve).

of electromagnetic fields. One reason for this is that to date there has not been complete agreement about the choice of the Hermitian operator for the phase (see, e.g., Refs. 21–23). Here we skirt around this problem by treating the phase fluctuations through analysis of the dispersion in the fluctuations of the phase-dependent quadrature of components of the radiation modes ω_j:

$$X_j^{\theta_j}=a_je^{-i\theta_j}+a_j^+e^{i\theta_j},\quad (j=0,1,2). \tag{31}$$

From the standpoint of the measurement problem this approach corresponds to designing the phase-sensitive experiments by the homodyning technique, in which the phase θ_j of the reference waves can be varied relative to the phase ψ_j^0 of the mode being studied.

As is well known, the quantum-statistical dispersion of the fluctuations of the quadrature components (31) can be expressed in terms of the phase fluctuations of the modes in the P representation for $\theta_j-\psi_j^0=\pi/2$ (Ref. 10):

$$\langle(\Delta X_j^{\pi/2})^2\rangle=1+4n_j^0\langle\delta\psi_j(t)^2\rangle. \tag{32}$$

To analyze the fluctuation correlations we consider the variance of the sum or difference of the quadrature components of the modes ω_i and ω_j in the cavity:

$$V_{ij}^{(\pm)}(\theta_i,\theta_j)=\langle[\Delta(X_i^{\theta_i}\pm X_j^{\theta_j})]^2\rangle. \tag{33}$$

The commutation relations for the two canonically conjugate quadrature operators with definite phases $\theta_{i,j}$ and $\theta_{i,j}+\pi/2$ are equal to

$$[X_i^{\theta_i}\pm X_j^{\theta_j},X_i^{\theta_i+\pi/2}\pm X_j^{\theta_j+\pi/2}]=4i,\quad (i\neq j), \tag{34}$$

from which it follows in the general case of arbitrary θ_i,θ_j that the conditions for squeezing of the fluctuations are

$$V_{ij}^{(\pm)}(\theta_i,\theta_j)<2. \tag{35}$$

For coherent states of the electromagnetic field the fluctuation level is equal to $V_{ij}^{(\pm)}(\theta_i,\theta_j)=2$.

It is not difficult to show that in the linearized theory we are using the variance (33) is

$$\begin{aligned}V_{ij}^{(\pm)}(\theta_i,\theta_j)=2+\Big\langle\Big[&\frac{\delta n_i(t)}{\sqrt{n_i^0}}\cos(\theta_i-\psi_i^0)\\ &\pm\frac{\delta n_j(t)}{\sqrt{n_j^0}}\cos(\theta_j-\psi_j^0)\Big]^2\Big\rangle\\ &+4\langle[\sqrt{n_i^0}\delta\psi_i(t)\sin(\theta_i-\psi_i^0)\\ &\pm\sqrt{n_j^0}\delta\psi_j(t)\sin(\theta_j-\psi_j^0)]^2\rangle,\end{aligned} \tag{36}$$

and for $\theta_{i,j}-\psi_{i,j}^0=\pi/2$ depends only on the phase fluctuations

$$\begin{aligned}V_{ij}^{(\pm)}(\pi/2)&=2+4\langle[\sqrt{n_i^0}\delta\psi_i(t)\pm\sqrt{n_j^0}\delta\psi_j(t)]^2\rangle\\ &=2+4n_i^0\langle\delta\psi_i(t)^2\rangle+4n_j^0\langle\delta\psi_j(t)^2\rangle\\ &\pm8\sqrt{n_i^0n_j^0}\langle\delta\psi_i(t)\delta\psi_j(t)\rangle.\end{aligned} \tag{37}$$

[Here and in what follows we use the notation $V_{ij}^{(\pm)}=(\theta_i,\theta_j)\equiv V_{ij}^{(\pm)}(\theta)$ for the phases $\theta_i-\psi_i^0=\theta_j-\psi_j^0\equiv\theta$]. From (35) and (37) we find conditions for the suppression of the fluctuations of the sum and difference of the phases of two modes. In order to evaluate the quantities (37) we must turn to expressions (A2) and (A4). Here we give the final results.

In the region $1<\varepsilon<2$, where we have $n_1^0=n_2^0=\gamma_0/\chi$ for the pump modes, we find

$$V_{12}^{(-)}(\pi/2)=2-2(\varepsilon-1)/\varepsilon, \tag{38}$$

$$V_{12}^{(+)}(\pi/2)=2+(\varepsilon-1)/(1+r-\varepsilon/2). \tag{39}$$

It is easy to see that the fluctuations are squeezed for the quadrature difference. The mean-square dispersion of the phase difference is a negative quantity in the P representation, which implies suppression of the fluctuations of the phase difference of the pump modes below the coherent level:

$$\langle[\delta(\psi_1-\psi_2)]^2\rangle=-\frac{\chi}{2\gamma_0}\cdot\frac{\varepsilon-1}{\varepsilon}. \tag{40}$$

This squeezing effect is the result of a strong positive correlation between the instantaneous phase fluctuations,

$$\langle\delta\psi_1(t)\delta\psi_2(t)\rangle=\frac{\chi}{8\gamma_0}\frac{(\varepsilon-1)(1+r)}{\varepsilon(1+r-\varepsilon/2)} \tag{41}$$

and vanishes for the sum of the quadratures or phases.

Similar results are found for combinations of the signal mode ω_0 and the pump modes $\omega_{1,2}$:

$$\begin{aligned}V_{01}^{(\pm)}(\pi/2)&=V_{02}^{(\pm)}(\pi/2)\\ &=\frac{3}{2}+\frac{(\varepsilon-1)(2\varepsilon-r-1)\pm\varepsilon\sqrt{2r(\varepsilon-1)}}{2\varepsilon(1+r-\varepsilon/2)}.\end{aligned} \tag{42}$$

In the region $\varepsilon>2$ we can obtain analytical results under the conditions $\varepsilon^2\gg1$ and $r\varepsilon^2\gg1$. However, in this case the contributions of the intermode correlations of the phase fluctuations [the last terms in Eq. (37)] are found to be small in comparison with the other terms, i.e., squeezing in $V_{ij}^{(\pm)}(\pi/2)$ is determined by the sum $4n_i^0\langle\delta\psi_i^2\rangle + 4n_j^0\langle\delta\psi_j^2\rangle$ or by the dispersion of the fluctuations of each of the quadratures (32).

When the phase of the quadrature components is taken to be $\theta_{i,j}-\psi_{i,j}^0=0$, determining the amplitude fluctuations of the modes, the dispersion (36) depends only on the fluctuations of the photon number:

$$V_{ij}^{(\pm)}(0)=2+\langle\delta n_i(t)^2\rangle/n_i^0+\langle\delta n_j(t)^2\rangle/n_j^0 \pm 2\langle\delta n_i(t)\delta n_j(t)\rangle/\sqrt{n_i^0 n_j^0}. \quad (43)$$

Expressions for the correlation functions $\langle\delta n_i(t)\delta n_j(t)\rangle$ $(i,j=0,1,2)$ can be obtained using Eqs. (A1) and (A3). We present a final result for the case of the greatest squeezing of the fluctuations of the quadrature sum of the pump modes in the region $1<\varepsilon<2$:

$$V_{12}^{(+)}(0)=2-2(\varepsilon-r-1)/\varepsilon. \quad (44)$$

The suppression of the fluctuations is greatest in the limit $\varepsilon\to 2$ and $r\ll 1$, and results from the negative correlation [see also Eq. (48)] between the instantaneous fluctuations $\langle\delta n_1(t)\delta n_2(t)\rangle<0$ of the photon number.

Note that these photon-number correlation effects can be described more conventionally using the second-order correlation functions

$$g_{ij}^{(2)}=\frac{\langle a_i^+(t)a_j^+(t)a_j(t)a_i(t)\rangle}{\langle a_i^+a_i\rangle\langle a_j^+a_j\rangle}, \quad (i,j=0,1,2). \quad (45)$$

To lowest order in the quantum fluctuations, this quantity is equal to

$$g_{ij}^{(2)}=1+\frac{\langle\delta n_i(t)\delta n_j(t)\rangle}{n_i^0 n_j^0}. \quad (46)$$

In using these expressions, however, we should keep in mind that the effects they describe, in accordance with the conditions for the applicability of the linearization technique

$$\sqrt{\langle(\delta n_i)^2\rangle}\gg n_i^0, \quad |\langle(\delta\psi_i)^2\rangle|\ll 1, \quad (47)$$

are very small. Hence the results for $g_{ij}^{(2)}$ have a restricted range of applicability and are useful only by virtue of their simplicity and to clarify the physics of the phenomena in question.

As shown by calculations using Eqs. (A1) and (A3), the correlations within photons of the different modes ω_i show a bunching behavior $g_{ii}^{(2)}>1$, while those between photons of different modes show an antibunching behavior $g_{ij}^{(2)}<1$ $(i\neq j)$. In particular, in the region $1<\varepsilon<2$ we find

$$g_{12}^{(2)}=1-\frac{\chi}{\gamma_0}\frac{\varepsilon-(1+r)(2-\varepsilon)}{2\varepsilon(2-\varepsilon)}, \quad (48)$$

$$g_{01}^{(2)}=g_{02}^{(2)}=1-\frac{\chi}{4\gamma(\varepsilon-1)}, \quad (49)$$

and antibunching in (48) occurs under the condition $r<2(\varepsilon-1)/(2-\varepsilon)$.

6. SPECTRA FOR THE FLUCTUATIONS OF THE SUM AND DIFFERENCE OF THE QUADRATURE COMPONENTS

We turn now to the fluctuation spectra for light at the output from the cavity in connection with the experimental measurement scheme using the twin homodyning technique.[13,14] In this method each of the two modes ω_i,ω_j $(i\neq j=0,1,2)$ of interest to us interferes with its own reference wave, as detected by a photodetector, and then the fluctuation spectrum of the sum or difference of the two photofluxes is measured. Such measurements make it possible to study the fluctuation spectrum of the sum or difference of the quadrature components of the fields at the frequencies ω_i and ω_j:

$$\begin{aligned}S_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)&=\int_{-\infty}^{+\infty}d\tau\, e^{i\omega\tau}\langle[Y_i^{\theta_i}(t)\pm Y_j^{\theta_j}(t)],\\&\quad[Y_i^{\theta_i}(t+\tau)\pm Y_j^{\theta_j}(t+\tau)]\rangle\\&=2+\int_{-\infty}^{+\infty}d\tau\, e^{i\omega\tau}\{\langle:Y_i^{\theta_i}(t),Y_i^{\theta_i}(t+\tau):\rangle\\&\quad+\langle:Y_j^{\theta_j}(t),Y_j^{\theta_j}(t+\tau):\rangle\\&\quad\pm[\langle Y_i^{\theta_i}(t),Y_j^{\theta_j}(t+\tau)\rangle+\langle Y_j^{\theta_j}(t),\\&\quad Y_i^{\theta_i}(t+\tau)\rangle]\}.\end{aligned} \quad (50)$$

Here

$$Y_k^{\theta_k}(t)=b_k(t)e^{-i\theta_k}+b_k^+(t)e^{i\theta_k}, \quad (k=i,j), \quad (51)$$

are the phase-dependent operators of the quadrature components of the fields at the output from the cavity near the frequencies ω_k, and the colons denote normal-ordered operators.

The first term in expression (50) is the vacuum level of the corresponding quantum fluctuations of the electromagnetic field. The quantum fluctuations in the sum or difference of the quadratures are squeezed below the vacuum level for $S_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)<2$, where the maximum effect (100% squeezing) corresponds to $S_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)=0$. The quantity (50) serves as a measure of the correlation between two quadrature components, and in this connection has a direct relation to the ideas involved in performing nondestructive quantum measurements[14,24–26] and demonstrating Einstein–Podolsky–Rosen correlations (see Refs. 14, 27, and 28, and the earlier work cited therein). Other practical applications may apply to ultrahigh-resolution spectroscopy[2] and control of quantum noise.[29–31]

In the linearized theory of fluctuations the result for the quantity (50), normalized to the vacuum noise level $S_0=2$, can be given in the following form:

$$S_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)/S_0=1+F_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)+N_{ij}^{(\pm)}(\theta_i,\theta_j,\omega), \quad (52)$$

where the quantity $F_{ij}^{(\pm)}$ is related to the fluctuations in the phases of the sum and difference modes:

$$F_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)=4\langle[\sqrt{\gamma_i n_i^0}\delta\psi_i(-\omega)\sin(\theta_i-\psi_i^0)$$

$$\pm \sqrt{\gamma_j n_j^0}\delta\psi_j(-\omega)\sin(\theta_j-\psi_j^0)]$$
$$\times[\sqrt{\gamma_i n_i^0}\delta\psi_i(\omega)\sin(\theta_i-\psi_i^0)$$
$$\pm \sqrt{\gamma_j n_j^0}\delta\psi_j(\omega)\sin(\theta_j-\psi_j^0)]\rangle, \quad (53)$$

and the quantity $N_{ij}^{(\pm)}$ can be expressed in terms of the photon-number fluctuations:

$$N_{ij}^{(\pm)}(\theta_i,\theta_j,\omega)=\left\langle\left[\left(\frac{\gamma_i}{n_i^0}\right)^{1/2}\delta n_i(-\omega)\cos(\theta_i-\psi_i^0)\right.\right.$$
$$\left.\pm\left(\frac{\gamma_j}{n_j^0}\right)^{1/2}\delta n_j(-\omega)\cos(\theta_j-\psi_j^0)\right]$$
$$\times\left[\left(\frac{\gamma_i}{n_i^0}\right)^{1/2}\delta n_i(\omega)\cos(\theta_i-\psi_i^0)\right.$$
$$\left.\left.\pm\left(\frac{\gamma_j}{n_j^0}\right)^{1/2}\delta n_j(\omega)\cos(\theta_j-\psi_j^0)\right]\right\rangle. \quad (54)$$

From these expressions we see that by appropriate choice of the phases θ_i and θ_j we can ensure that the spectrum (52) is expressed either in terms of phase fluctuations or photon-number fluctuations. As was already pointed out, this property of the process we are considering is ensured by the definiteness of the stationary phases of all three modes in the above-threshold generation regime, and is responsible for a significant departure from the familiar results[1,2,5] for nondegenerate parametric light generation and four-wave mixing with a single pump mode, where phase diffusion occurs.

7. SPECTRA OF SQUEEZED QUADRATURE DIFFERENCES: THE PHASE EFFECT

We give an expression for the fluctuation spectrum (52) in the case $\theta_i-\psi_i^0=\theta_j-\psi_j^0=\pi/2$, when it depends only on the phase fluctuations of the modes ω_i and ω_j $(i\neq j=0,1,2)$:

$$S_{ij}^{(\pm)}(\pi/2,\omega)/S_0=1+4\gamma_i n_i^0\langle\delta\psi_i(-\omega)\delta\psi_i(\omega)\rangle$$
$$+4\gamma_j n_j^0\langle\delta\psi_j(-\omega)\delta\psi_j(\omega)\rangle$$
$$\pm 8\sqrt{\gamma_i\gamma_j n_i^0 n_j^0}\,\mathrm{Re}\langle\delta\psi_i(-\omega)\delta\psi_j(\omega)\rangle. \quad (55)$$

The results for the phase fluctuation autocorrelation functions are given by Eqs. (25). The intermode correlation functions

$$C_{ij}(\omega)=8\sqrt{\gamma_i\gamma_j n_i^0 n_j^0}\,\mathrm{Re}\langle\delta\psi_i(-\omega)\delta\psi_j(\omega)\rangle \quad (56)$$

can be calculated using the solutions of Eqs. (5) with the noise correlations (8).

We first give the result for the case in which the two combining modes are the pump modes ω_1 and ω_2:

$$C_{12}(\omega)=\frac{4\sqrt{q}}{r^2\bar{d}(\omega)}\{(\omega/\gamma)^4+[1+q+2r^2-2rp]$$
$$\times(\omega/\gamma)^2+2r^2\varepsilon^2\}. \quad (57)$$

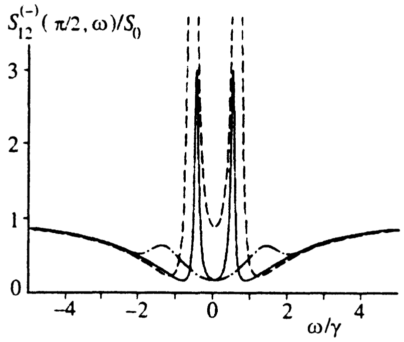

FIG. 3. Spectral curves of the fluctuations $S_{12}^{(-1)}(\pi/2,\omega)/S_0$ of the difference in the quadratures of the pump modes as a function of ω/γ: $\varepsilon=2.2$, $r=0.5$ (chain); $\varepsilon=2.2$, $r=0.05$ (solid); and $\varepsilon=6$, $r=0.01$ (dashed curve).

Analysis of the general expression (55) for this case shows that the squeezing is larger for the fluctuations of the difference spectra. Calculations reveal that in the region $1<\varepsilon<2$ the expression for the difference spectrum simplifies greatly and reduces to the following:

$$S_{12}^{(-)}(\pi/2,\omega)/S_0=1-\frac{4(\varepsilon-1)}{\varepsilon^2+(\omega/\gamma)^2}. \quad (58)$$

This expression does not depend on $r=\gamma_0/\gamma$ and shows that 100% squeezing of the difference fluctuations of the quadratures is possible in the limit $\varepsilon\to 2$ in the low-frequency range $\omega\ll\gamma$ of the spectrum.

In the range $\varepsilon>2$ the form of the spectrum $S_{12}^{(-)}(\pi/2,\omega)/S_0$ becomes more complicated. The results of the calculations are plotted in Fig. 3 for different values of ε and r. Squeezing decreases with increasing ε in the low-frequency range and disappears for $\varepsilon\gg 2$. However, for fixed values of ε and small r, two symmetric minima appear in the sideband regions of the spectrum. The amount of fluctuation suppression at these sideband frequencies exceeds the squeezing at zero frequency and approaches total squeezing in the limit $r\to 0$. Note that there is no squeezing of the quadratures of either of the pump modes for small r and relatively large values of ε, and the contribution of the autocorrelation functions of the phase fluctuations to the magnitude of the squeezing of the difference of these quadratures is vanishingly small. The difference compression (see, e.g., the trace for $\varepsilon=6$ and $r=0.01$ in Fig. 3) is entirely determined by the strong positive correlation between phase fluctuations of the pump modes.

For the case in which the mode ω_0 and one of the pump modes $\omega_k (k=1,2)$ combine, the result for the cross-correlation function $C_{0k}(\omega)$ is

$$C_{0k}(\omega)=\frac{8}{\sqrt{2q_k}\bar{d}(\omega)}\{[q+q_k](\omega/\gamma)^2+2q\varepsilon^2$$
$$+(1-q)(q+q_k)\}. \quad (59)$$

The quantity $C_{0k}(\omega)$ is positive for arbitrary values of ε and r. Consequently, just as in the case of the pump modes, effective squeezing of the fluctuations should be expected in the difference spectrum $S_{12}^{(-)}(\pi/2,\omega)/S_0$.

Figure 3 shows typical traces of the $S_{12}^{(-)}(\pi/2,\omega)/S_0$ spectrum as a function of $\omega/\sqrt{\gamma_0\gamma}$. The behavior of the functions for different ε and r is complicated, and the values of ε and r used were chosen to maximize the contribution of the intermode correlation functions $C_{0k}(\omega)$ to the squeezing of the quadratures of ω_0 and ω_k. With an appropriate choice of the parameter r, the magnitude of the squeezing that can actually be achieved may be at least 50% over the entire above-threshold region $\varepsilon>1$ at the zero or sideband frequencies of the spectrum. The maximum squeezing occurs close to $\varepsilon=1.2$ and is equal to $\sim 83\%$.

8. SQUEEZING OF THE SUM OF THE QUADRATURES: INTENSITY CORRELATION

As already noted in Sec. 5, the present approach enables us to address the questions of intensity correlations of two modes from the standpoint of phase-dependent measurements. For phases $\theta_i-\psi_i^0=\theta_j-\psi_j^0=0$ the spectrum (52) is determined by the fluctuations of the photon numbers of the modes ω_i and ω_j. As it turned out, this quantity is related to another experimentally measurable quantity, the spectrum of fluctuations of the sum or difference of the intensities of the two modes,

$$P_{ij}^{(\pm)}(\omega)=\int_{-\infty}^{+\infty} d\tau e^{i\omega\tau}\langle (N_i(t)\pm N_j(t)),(N_i(t+\tau)\pm N_j(t+\tau))\rangle, \tag{60}$$

where $N_i(t)=b_i^+(t)b_i(t)$ are the photon-number operators. It can easily be shown that to lowest order in the fluctuations, this quantity is

$$P_{ij}^{(\pm)}(\omega)=P_0\left\{1+\frac{\gamma_i}{n_i^0}\langle\delta n_i(-\omega)\delta n_i(\omega)\rangle+\frac{\gamma_j}{n_j^0}\langle\delta n_j(-\omega)\delta n_j(\omega)\rangle\pm 2\left(\frac{\gamma_i\gamma_j}{n_i^0n_j^0}\right)^{1/2}\times\mathrm{Re}\langle\delta n_i(-\omega)\delta n_j(\omega)\rangle\right\}, \tag{61}$$

where

$$P_0=\langle N_i\rangle+\langle N_j\rangle=|b_i^0|^2+|b_j^0|^2 \tag{62}$$

is the coherent fluctuation level of two modes, which is the same for the sum and difference of the intensities. Comparison of the results (52) for $\theta_i-\psi_i^0=\theta_j-\psi_j^0=0$ and (61) shows that the normalized spectra are identical:

$$S_{ij}^{(\pm)}(0,\omega)/S_0=P_{ij}^{(\pm)}(\omega)/P_0. \tag{63}$$

The spectra $P_{ij}^{(\pm)}(\omega)$ are usually measured (see, e.g., Refs. 7 and 8) by direct detection of the intensities of the two modes. This relation shows that for the case of coupled modes with definite phases they can also be measured by the homodyning technique with two reference fields.

We have shown[10,12] that substantial (close to total) compression of the quantum fluctuations in the sum of the intensities of the pump modes below the coherent level is possible: $P_{12}^{(+)}(\omega)/P_0<1$. This effect follows from the anticorrelation between the fluctuations of the photon numbers of these modes. It is clear that by virtue of (63) the effect will manifest itself also in the squeezing of fluctuations of the quadrature sum $S_{12}^{(+)}(0,\omega)/S_0<1$.

For completeness it is also of interest to consider the case in which the combining modes consist of ω_0 and one of the pump modes ω_k ($k=1,2$). In this case the result (63) for the spectra can be obtained using Eq. (18) and a calculation of the corresponding cross-correlation function. Using the solutions of Eqs. (4) and the noise correlations (7) for this function we find

$$2\left(\frac{\gamma_0\gamma}{n_0^0n_k^0}\right)^{1/2}\mathrm{Re}\langle\delta n_0(-\omega)\delta n_k(\omega)\rangle=-\frac{4\sqrt{2q_k}}{d(\omega)}\{(1+q_n)(\omega/\gamma)^2+(1-q)(1-q_n)-2(q-q_n^2)\},\quad (n\neq k=1,2). \tag{64}$$

Analysis of this result shows that in the region $1<\varepsilon<2$ the correlation between the photon-number fluctuations of these modes is negative, while in the region $\varepsilon>2$ it can be either positive or negative, depending on the values of ε and r. In the region $\varepsilon>2$, where $n_1^0\neq n_2^0$ holds, the results also depend on which of the pump modes ω_1, ω_2 combines with ω_0. However, in any case this correlation is found not to be so strong that it can cancel the excess (relative to the coherent) noise level resulting from the contribution of the autocorrelation functions $\langle\delta n_0(-\omega)\delta n_0(\omega)\rangle$ and $\langle\delta n_k(-\omega)\delta n_k(\omega)\rangle$. Consequently, the greatest suppression of the fluctuations is found to occur only in the sum of the quadratures and in the sum of the intensities of ω_0 and that pump mode whose intensity in the cavity increases in the region $\varepsilon>2$. The magnitude of this suppression is $\sim 17\%$ for $\varepsilon\approx 9$ at the zero frequency of the spectrum ($\omega=0$).

APPENDIX

We present in general form the results for the two-time correlation functions $\langle\delta n_i(t)\delta n_j(t')\rangle$ and $\langle\delta\psi_i(t)\delta\psi_j(t')\rangle$ ($i,j=0,1,2$). They can be represented as elements of the matrices $\langle\delta n(t)\delta n(t')^T\rangle_{\alpha\beta}$ and $\langle\delta\psi(t)\delta\psi(t')^T\rangle_{\alpha\beta}$, ($\alpha=i+1,\beta=j+1$). Using the general form of the solutions of Eqs. (4) and (5) written in the time representation[10] and the noise correlation functions (7) and (8), we derive them in the form

$$\langle\delta n(t)\delta n(t')^T\rangle=\sum_{m=1}^{3}K_m\exp(-\lambda_m|t-t'|), \tag{A1}$$

$$\langle\delta\psi(t)\delta\psi(t')^T\rangle=\sum_{m=1}^{3}\bar{K}_m\exp(-\bar{\lambda}_m|t-t'|). \tag{A2}$$

Here K_m and $\bar{K}_m$ are matrices with elements $K_{m,\alpha\beta}$ and $\bar{K}_{m,\alpha\beta}$, defined as follows:

$$K_m = \sum_{n=1}^{3} \frac{\Pi_{i\neq m}(A-\lambda_i I)D[\Pi_{j\neq n}(A-\lambda_j I)]^T}{(\lambda_m+\lambda_n)\Pi_{i\neq m}(\lambda_m-\lambda_i)\Pi_{j\neq n}(\lambda_n-\lambda_j)}, \quad \text{(A3)}$$

$$\bar{K}_m = \sum_{n=1}^{3} \frac{\Pi_{i\neq m}(\bar{A}-\bar{\lambda}_i I)\bar{D}[\Pi_{j\neq n}(\bar{A}-\bar{\lambda}_j I)]^T}{(\bar{\lambda}_m+\bar{\lambda}_n)\Pi_{i\neq m}(\bar{\lambda}_m-\bar{\lambda}_i)\Pi_{j\neq n}(\bar{\lambda}_n-\bar{\lambda}_j)}, \quad \text{(A4)}$$

$\lambda_m, \bar{\lambda}_m$ $(m=1,2,3)$ are the eigenvalues of the matrices A and $\bar{A}$ given by Eq. (6) respectively, D, $\bar{D}$ are the diffusion matrices introduced according to

$$\langle F(t)F(t')^T\rangle = D\delta(t-t'),$$

$$\langle f(t)f(t')^T\rangle = \bar{D}\delta(t-t'),$$

and equal to

$$D=\begin{pmatrix} 2\gamma_0 n_0^0 & 0 & 0 \\ 0 & 0 & -\gamma_0 n_0^0 \\ 0 & -\gamma_0 n_0^0 & 0 \end{pmatrix},$$

$$\bar{D}=\begin{bmatrix} -\frac{\gamma_0}{2n_0^0} & 0 & 0 \\ 0 & 0 & \frac{\chi^2 n_0^0}{4\gamma_0} \\ 0 & \frac{\chi^2 n_0^0}{4\gamma_0} & 0 \end{bmatrix}. \quad \text{(A5)}$$

In the region $1<\varepsilon<2$ the eigenvalues λ_m and $\bar{\lambda}_m$ are equal to

$$\lambda_{1,2}=\frac{1}{2}(\gamma\varepsilon \pm \sqrt{\gamma^2\varepsilon^2-16\gamma_0\gamma(\varepsilon-1)}) \quad \lambda_3=\gamma(2-\varepsilon);$$

$$\bar{\lambda}_{1,2}=\frac{1}{2}(2\gamma_0+2\gamma-\gamma\varepsilon \pm \sqrt{(2\gamma_0+2\gamma-\gamma\varepsilon)^2-8\gamma_0\gamma\varepsilon}) \quad \bar{\lambda}_3=\gamma\varepsilon. \quad \text{(A6)}$$

In the region $\varepsilon>2$ we can only give their approximate values for $\varepsilon^2 \gg 1$ and $r\varepsilon^2 \gg 1$:

$$\lambda_{1,2}=\frac{1}{2}(\gamma \pm \sqrt{\gamma^2-8\gamma_0\gamma\varepsilon^2}) \quad \lambda_3=\gamma;$$

$$\bar{\lambda}_{1,2}=(\gamma_0+\gamma/2)\pm\sqrt{(\gamma_0+\gamma/2)^2-2\gamma_0\gamma\varepsilon^2}, \quad \bar{\lambda}_3=\gamma. \quad \text{(A7)}$$

If the eigenvalues $\lambda_{1,2}$ and $\bar{\lambda}_{1,2}$ are complex, then we have $\lambda_2 = \lambda_1^*$, $\bar{\lambda}_2 = \bar{\lambda}_1^*$ and we can write $\lambda_{1,2} = \lambda' \pm i\lambda''$, $\bar{\lambda}_{1,2} = \bar{\lambda}' \pm i\bar{\lambda}''$. In this case it can easily be shown that the matrices K_3 and $\bar{K}_3$ are real, and the matrices $K_{1,2}$ and $\bar{K}_{1,2}$ satisfy the relations $K_2 = K_1^*$, $\bar{K}_2 = \bar{K}_1^*$. Accordingly, it also makes sense to introduce the real and imaginary parts of the matrices $K_{1,2}$ and $\bar{K}_{1,2}$:

$$K_{1,2}=K' \pm iK'', \quad \bar{K}_{1,2}=\bar{K}' \pm i\bar{K}''. \quad \text{(A8)}$$

Using this notation we derive Eqs. (27) and (28) from the relations (A3) and (A4).

[1)] Nonclassical optical effects, but for the case of a resonant monatomic medium, were treated in Refs. 16 and 17 without taking pump depletion into account.

[2)] Here they are given in a more compact form than in Ref. 10.

[3)] This remark applies equally to the analysis of the intensity spectrum given in Sec. 3.

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Translated by David L. Book