PHYS2100: Hamiltonian mechanics assignment: part 2
Due 5pm Friday 27th October 2006.

5. The "Matt map" is defined by

$$x_{n+1} = \mu x_n(1 - x_n^2).$$

(a) Find the upper limit to μ such that x remains bounded.

(b) Find the fixed points, and the range of μ for which they are stable.

6. The logistic map is defined by

$$x_{n+1} = \mu x_n(1 - x_n).$$

(a) Show that there is a fixed point at $x_f = 1 - 1/\mu$.

(b) We have seen that the fixed point in (a) is only stable for $\mu < 3$, beyond which the map undergoes a bifurcation. We can find the value for μ that the next bifurcation occurs using slightly more complicated analysis. The values of x for the map undergoes a period two oscillation can be found by solving

$$f^2(x) = x.$$

Show that this results in the equation

$$\mu x^3 - 2\mu x^2 + (1 + \mu)x + \left(1 - \frac{1}{\mu^2}\right) = 0.$$

(c) From part (a) we have already know one of the roots of this cubic equation. Hence factorise your answer from (b), and find the values of x for the stable period two oscillation by solving the remaining quadratic.

(d) The stability criterion for a fixed point in a 1D map can be generalised to determine the stability of a p-cycle: for a p-cycle from $x_1, x_2, \ldots, x_p$, the cycle is stable if

$$\prod_{j=1}^{p} |f'(x_j)| < 1.$$

By applying this criterion, show that the two-cycle for the logistic map is stable for $3 < \mu < 1 + \sqrt{6}$.

7. Investigate the Poincarè sections of the driven pendulum using the matlab code provided on the course website. There will be time for this in the computer lab 78-336 on Wednesday 25th October from 2–4pm.

(a) Set the frequency to be 0.9, and look at how the Poincarè section changes as you increase the amplitude of the driving from zero . Describe qualitatively what you find, noting particularly the appearance/disappearance of any islands. Sketch qualitatively (or print out) two or three examples of what you see.

(b) Choose another driving frequency showing different qualitative behaviour as you increase the amplitude of the driving. Again describe what you find and sketch or print examples of the Poincarè sections.