1. Prove that for any function $f(q, p, t)$ then
\[ \dot{f} = \{f, H\} + \frac{\partial f}{\partial t}. \]

2. Show that if $\{Q, P\} = 1$ then
\[ \dot{P} = -\frac{\partial \bar{H}}{\partial Q} \]
where $\bar{H}(Q, P) = H(q, p)$.

3. Show that the following transformation is canonical
\[ Q = e^{\lambda}(q \cos \theta + p \sin \theta), \]
\[ P = e^{-\lambda}(-q \sin \theta + p \cos \theta). \]

4. Show that the area enclosed by the separatrix of the vertical pendulum with Hamiltonian
\[ H = \frac{l^2}{2} - \alpha^2 \cos \theta, \]
is $16\alpha$. Deduce that the maximum value of the action for librating motion is $8\alpha/\pi$.

5. A particle of mass $m$ experiences the potential
\[ V(\psi) = A\psi, \quad (0 \leq \psi \leq \alpha), \]
\[ V(\psi) = A\alpha, \quad (\alpha \leq \psi \leq \pi), \]
\[ V(\psi) = V(-\psi), \]
which is defined to be periodic outside the range $(-\pi \leq \psi \leq \pi)$.

(a) Sketch a graph of this potential, and draw the phase portrait for the system. What is the energy that separates the librations and rotations?

(b) Find the action-angle variables for each type of motion, and determine the frequency $\omega$ of motion for a trajectory of energy $E$. Sketch a graph of the frequency $\omega$ versus $E$. 

** ≡ To be handed in.