PHYS2100: Hamiltonian mechanics tutorial sheet 3
Problem 1: Due 5pm Friday 20th October 2006.

** $\equiv$ To be handed in.

** 1. The tent map is defined as

$$x_{n+1} = \mu \left(1 - 2 \left| x_n - \frac{1}{2} \right| \right).$$

(a) What are the limits on μ if we must have $0 \leq x_n \leq 1$?

(b) Draw a graphical representation of the tent map for $\mu = 0.75$ for two different initial points, and five iterations.

(c) Find the fixed points of the tent map, and determine their stability.

(d) Find the Lyapunov exponent for the tent map. For what range of μ is this map chaotic?

(e) (Optional) Plot a bifurcation diagram for the tent map.

2. Investigate the sine map (numerically if you have to):

$$x_{n+1} = \mu \sin(\pi x_n).$$

(a) What are the limits on μ?

(b) What are the fixed points, and where are they stable?

(c) For what range of μ is the map chaotic?

3. The dynamics of a particular particle are described by the Hamiltonian

$$H = \frac{p^2}{2} + \frac{1}{2} \tanh^2 q.$$

Show that the frequency of motion is given by

$$\omega(H) = \sqrt{1 - 2H}.$$

Hint:

$$\int dx \frac{(a^2 - x^2)^{1/2}}{1 - x^2} = \sin^{-1}\left(\frac{x}{a}\right) - (1 - a^2)^{1/2} \tan^{-1}\left[\frac{x(1 - a^2)^{1/2}}{(s^2 - x^2)^{1/2}}\right].$$