Questions 2 and 7 are due Fri 4th August by 5pm.

PHYS2100 Problem Sheet 1

Semester 2, 2006
Problems marked † are harder than the rest.

1. A particle moves towards a centre of attraction starting from rest a distance $a$ from the centre. If its velocity at a distance $x$ from the centre varies as ($\alpha$ a constant)

$$\alpha \left(\frac{a^2 - x^2}{x^2}\right)^{1/2},$$

determine $x$ as a function of $t$.

2. A particle of mass $m$ is acted on by a force

$$m\mu(x + a^4/x^3)$$
towards the origin. Here $x$ denotes the distance of the particle from the origin and $\mu, a$ are positive constants.

(i) Show that the motion is conservative and find the corresponding potential energy.

(ii) Assuming that the particle starts at rest a distance $a$ from the origin, show that the particle's velocity is given by

$$v = -\sqrt{\mu} \left(\frac{a^4 - x^4}{x^2}\right)^{1/2}.$$

(iii) † Show that the particle will arrive at the origin at time $\pi/(4\sqrt{\mu})$.

3. Calculate the escape velocity from earth, given that the radius of the earth is $6.378 \times 10^6$ m.

4. A star is attracted along a straight line towards a black hole with Schwarzschild radius $R$. Given that the velocity of the star at an infinite distance from the black hole is zero, show that the velocity of the star varies with its distance $x$ from the centre of the black hole according to

$$v = -c\sqrt{\frac{R}{x}}$$

where $c$ is the speed of light.

5. Show that a particle in 2-space which moves so that it its position vector is always orthogonal to its velocity vector, must move in a circular path.
6. Find the length of arc for the following curves between $t = 0$ and $t = 1$:

(i) $(x, y, z) = \left( \frac{1}{2} t, \frac{1}{3} (1 - t) \frac{2}{3}, \frac{1}{3} (1 + t) \frac{2}{3} \right)$
(ii) $(x, y, z) = (t \sin t + \cos t, \sin t - t \cos t, \frac{1}{3} t^3)$.

7. Copper tubing is wrapped in a circular helix around a cylindrical core that has a 12 cm diameter. What length of tubing will make one complete turn around the cylinder in a distance of 20 cm measured along the axis of the cylinder?

8. Determine the work done by the gravitational force

$$ F = -mg \hat{k} $$

on a particle with mass $m$ which moves along the expanding helix

$$(x, y, z) = (t \cos t, \sin t, t), \quad 0 \leq t \leq \frac{\pi}{2}.$$ 

9. Let $a(t), b(t)$ be vector functions in 3-space and $f(t)$ a scalar function of $t$. Prove the following:

(i) $\frac{d}{dt} (f(t)a) = \dot{f} a + f \dot{a}$
(ii) $\frac{d}{dt} (a \cdot b) = \dot{a} \cdot b + a \cdot \dot{b}$
(iii) $\frac{d}{dt} (a \times b) = \dot{a} \times b + a \times \dot{b}$. 

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