PHYS3020 Assignment 2 2007

Due 5pm Friday 1st June 2007.

You should get started on this now. Don’t leave it till the last couple of days otherwise you certainly won’t finish!

1. Schroeder: 7.29: Sommerfeld expansion

2. Schroeder: 7.33: Conduction band electrons


4. Schroeder: 7.63: 2D Debye solid

5. Trapped atomic gases. The next questions consider dilute atomic vapours confined in a trapping potential as realised in experiments, in particular in the basement of the Physics Annexe at UQ. Consider a gas of \( N \) non-interacting, non-relativistic identical atoms of mass \( m \), confined in a three-dimensional potential \( U(\mathbf{r}) \) that varies on a length scale much larger than the thermal de Broglie wavelength \( \lambda_T \). [Hint: make use of spherical polar coordinates for the integrals in this questions. A trigonometric transformation will allow you to do the integral in (b)]

(a) Given the definition that

\[
g(\epsilon) \approx \frac{1}{\hbar^3} \int d^3rd^3p \delta[\epsilon - H(\mathbf{r}, \mathbf{p})],
\]

where \( H(\mathbf{r}, \mathbf{p}) = p^2/2m + U(\mathbf{r}) \) show that the density of states can be written

\[
g(\epsilon) = 2\pi \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_V d^3r \sqrt{\epsilon - U(\mathbf{r})}.
\]

Write down the inequality that defines the volume \( V \) of the integral for the general case. From this expression, what is the density of states for a cube of side \( L \)?

(b) For an isotropic harmonic oscillator potential we have

\[
U(\mathbf{r}) = \frac{1}{2} m \omega^2 (x^2 + y^2 + z^2)
\]

Show that the density of states is

\[
g(\epsilon) = \frac{\epsilon^2}{2(\hbar\omega)^3}.
\]
6. **Trapped Bose gas.** Suppose that the temperature $T$ is sufficiently low that a macroscopic number of atoms occupy the ground state of the isotropic harmonic trap.

(a) Argue that the number of bosons in excited states is given by

$$ N_{ex}(T) = \int_0^\infty d\epsilon \ g(\epsilon) \frac{1}{\exp(\epsilon/k_B T) - 1}. $$

(b) Evaluate the integral of part (a) for the harmonic potential by making use of the expansion

$$ \frac{1}{1 - x} = \sum_n x^n, \text{ for } |x| < 1. $$

Express your results in terms of zeta functions and gamma functions, where

$$ \int_0^\infty x^{n-1} e^{-x} dx = \Gamma(n), \quad \zeta(n) = \sum_{m=1}^{\infty} \frac{1}{m^n}. $$

(c) Explain how the transition temperature $T_c$ for Bose-Einstein condensation can be determined from the expression in (c). Given $\Gamma(n) = (n - 1)!$ for integer $n$ and $\zeta(3) = 1.202$, show that for the isotropic harmonic oscillator the critical temperature for BEC is given by

$$ k_B T_c = 0.94 \frac{N^{1/3}}{\hbar \omega}. $$

What is the critical temperature for BEC for $3 \times 10^5$ atoms of $^{87}\text{Rb}$ ($Z=37$) in a trap with frequency $\omega = 2\pi \times 150$ Hz? (These are similar parameters to the experiment at UQ).

(f) For a general density of states we have $g(\epsilon) \propto \epsilon^{\nu}$. From your answer to (d), what condition must $\nu$ satisfy for Bose-Einstein condensation to occur? Explain your answer.